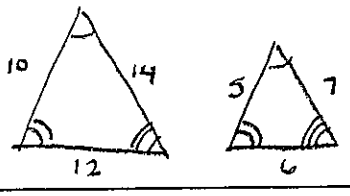
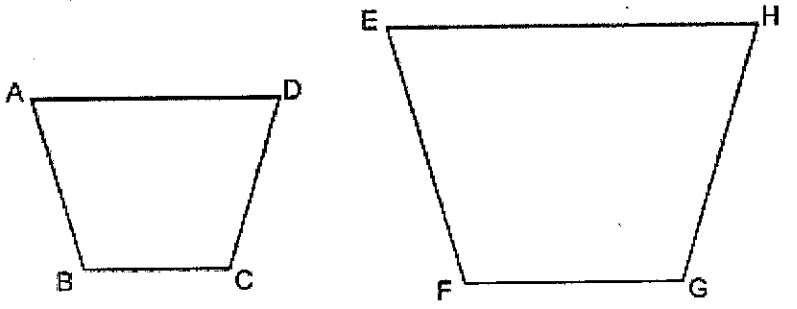
Vocabulary Unit 6

Key

Term	Definition	Picture
Similar Triangle	Triangles with $\cong$ $\measuredangle$ and proportional side lengths	
Ratio	A comparison of 2 #	1 : 2
Proportion	A comparison of 2 different ratios	$\frac{3}{5} = \frac{x}{10}$
Congruent Angles	Angles that have the same measure	$\measuredangle 60°$ $\measuredangle 60°$

Properties of Similar Polygons



All corresponding angles are the same measure.

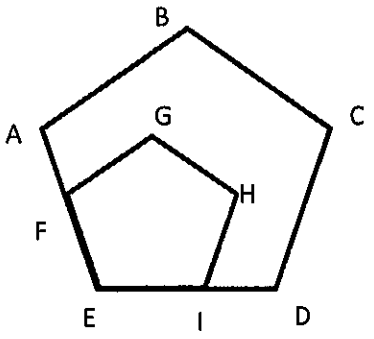
$\angle A = \angle E$
$\angle B = \angle F$
$\angle C = \angle G$
$\angle D = \angle H$

All corresponding sides are the same proportion.

$$\frac{AB}{EF} = \frac{BC}{FG} = \frac{CD}{GH} = \frac{DA}{HE}$$

Complete the congruence and proportion statements in the diagram below:

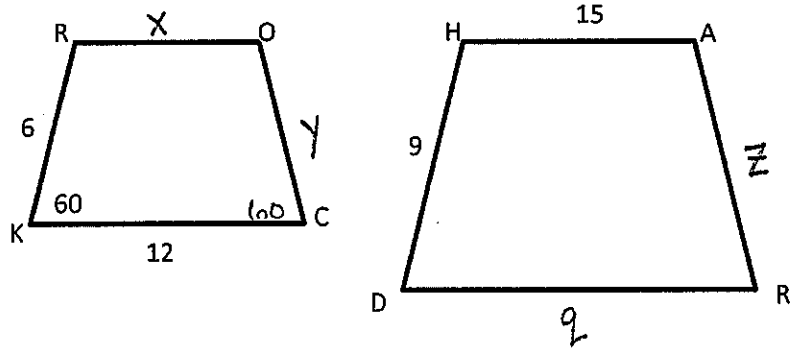
ABCDE~FGHIE



1.) $\measuredangle B \cong$ $\angle G$ 2.) $\measuredangle E \cong$ $\measuredangle E$ 3.) $\measuredangle D \cong$ $\measuredangle I$

4.) $\frac{AB}{FG} = \frac{BC}{GH}$ 5.) $\frac{AE}{FE} = \frac{ED}{EI}$ 6.) $\frac{IH}{DC} = \frac{EI}{ED}$

7.) Fill in **all of** the missing sides and angles in the **isosceles trapezoids** below:



ROCK~HARD

$\angle R = 120$ $\angle D = 60$

$\angle O = 120$ $\angle H = 120$

$\angle C = 60$ $\angle A = 120$

$\angle R = 60$

$\frac{6}{9} = \frac{12}{q}$ $DR = 18$

$6q = 108$

$\frac{6}{9} = \frac{x}{15}$ $RO = 10$

$9x = 90$

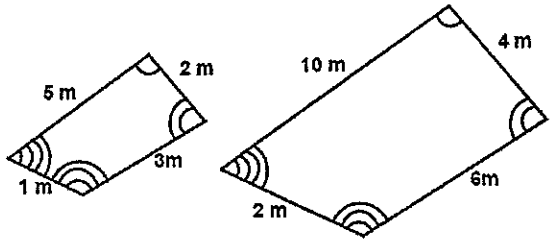
$OC = 6$ $AR = 9$

(isosceles trapezoid)

SOLVING FOR X IN SIMILAR TRIANGLES

State whether or not the polygons are similar. If they are then give the *similarity ratio*:

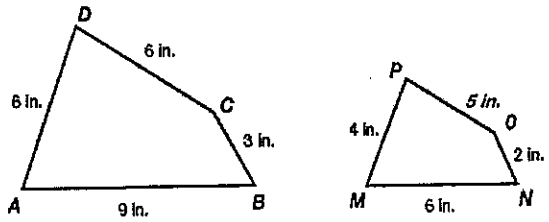
1.)



$\frac{5}{10} = \frac{2}{4} = \frac{3}{6} = \frac{1}{2}$

✓ yes 1:2

2.)

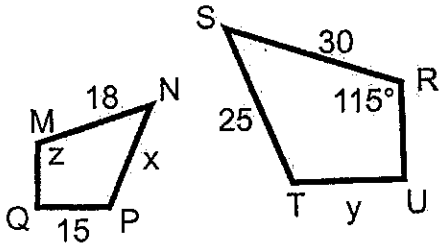


$\frac{6}{4} = \frac{9}{6} = \frac{6}{5} = \frac{3}{2}$

↑ NO!

Find the missing variables and state the similarity ratio:

3.) NMQP~SRUT



$\frac{30}{18} = \frac{y}{15}$

$450 = 18y$

$y = 25$

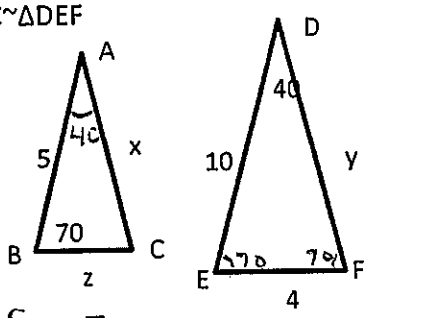
$xz = 115$

$\frac{30}{18} = \frac{25}{x}$

$30x = 450$

$x = 15$

4.) $\Delta ABC \sim \Delta DEF$



$\frac{5}{10} = \frac{z}{4}$

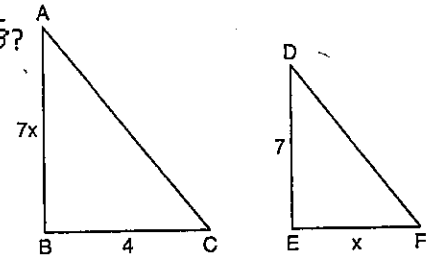
$z = 2$

$y = 10$

$x = 5$

5.)As shown in the diagram below, $\triangle ABC \sim \triangle DEF$, $AB = 7x$, $BC = 4$, $DE = 7$, and $EF = x$.

What is the length of $\overline{AB}$?



$\frac{7x}{7} = \frac{4}{x}$

$7x^2 = 28$

$x^2 = 4$

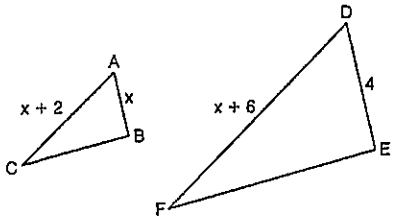
$x = \pm 2$

$x \neq -2$

$\boxed{x = 2}$

6.) In the diagram below, $\triangle ABC \sim \triangle DEF$, $DE = 4$, $AB = x$, $AC = x + 2$, and $DF = x + 6$.

Determine the length of $\overline{AB}$.



$\frac{4}{x} = \frac{x+6}{x+2}$

$x^2 + 6x = 4x + 8$

$x^2 + 2x - 8 = 0$

$(x + 4)(x - 2)$

$x \neq -4$

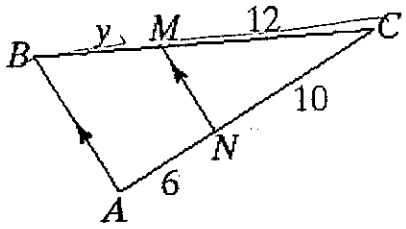
$\boxed{x = 2}$

SIDE SPLITTER THEOREM

If a line is **parallel** to one side of a triangle then we know that the base angles are ≅ and since the two triangles share an angle we know that the triangles are similar b/c of AA. Since the parallel line intersects the other two sides at the same rate the **sides** *(not the parallel lines)* split into equal parts.

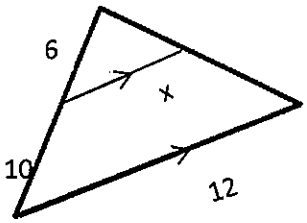
1.) **Using the Side-Splitter Theorem** Find y.

$$\frac{CM}{MB} = \frac{CN}{NA}$$



$\frac{12}{y} = \frac{10}{6}$ $\quad 72 = 10y$ $\quad y = 7.2$

2.) **What about this?**



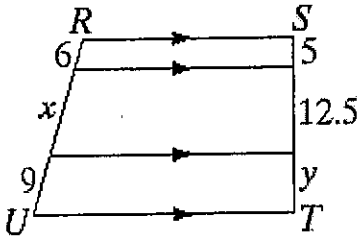
$\frac{6}{16} = \frac{x}{12}$ $\quad 72 = 16x$ $\quad x = 4.5$

Corollary to Side-Splitter Theorem

If three parallel lines intersect two transversals, then all corresponding parts are proportional.

Solve for x and y:

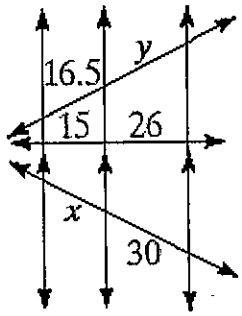
3.)



$\frac{6}{x} = \frac{5}{12.5}$ $\quad 75 = 5x$ $\quad x = 15$

$\frac{6}{9} = \frac{5}{y}$ $\quad 6y = 45$ $\quad y = 7.5$

4.)



$\frac{15}{26} = \frac{x}{30}$ $\quad 450 = 26x$ $\quad x = 17.3$

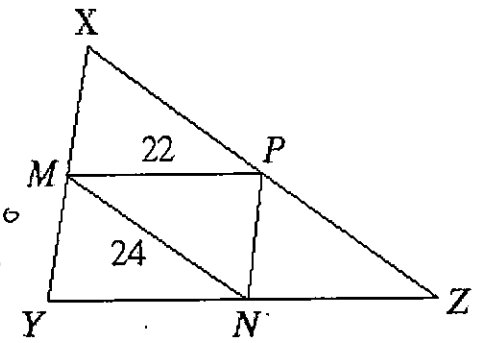
$\frac{16.5}{y} = \frac{15}{26}$ $\quad 429 = 15y$ $\quad y = 28.6$

MIDSEGMENT THEOREM

MIDSEGMENT: If a segment joins the midpoints of two sides of a triangle, then the segment is parallel to the third side and the *parallel line* is 1/2 its length.

Finding Lengths:

1.) In triangle XYZ, M, N, and P are midpoints.
The perimeter of triangle MNP 60.



$22 + 24 + PN = 60$
$46 + PN = 60$
$PN = 14$

Find XZ. $= 48$

Find YZ. 44

Find PN. 14

Find XY. 28

Find the perimeter of triangle XYZ.

$44 + 48 + 28 = 120$

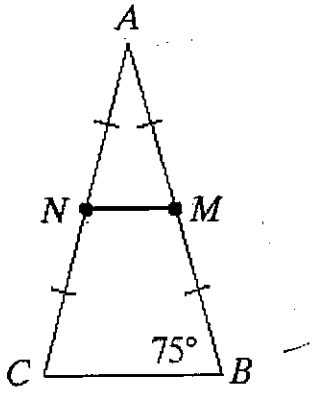
Parallel Segments:

2.) What types of angles are NAM and CAB? vertex

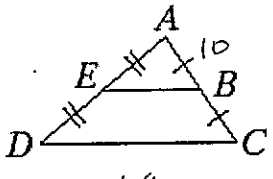
What is the measure of angle NMA? 75

What is the measure of angle ANM? Explain

75° isosceles Δ's



3.) $AB = 10$ and $CD = 18$. Find EB, BC, and AC.

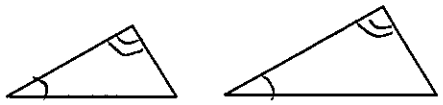


BC = 10
EB = 9
AC = 20

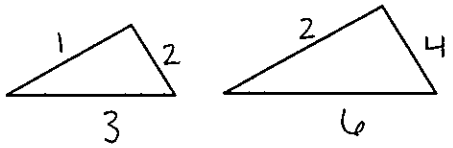
Notes: Similar Triangles

There are 3 ways you can prove triangles similar WITHOUT having to use all sides and angles.

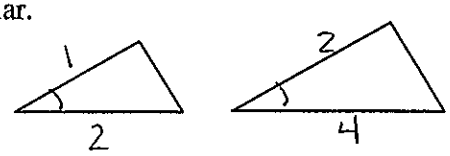
Angle- Angle Similarity (AA~) – If two angle of one triangle are congruent to two corresponding angles of another triangle, then the triangles are similar



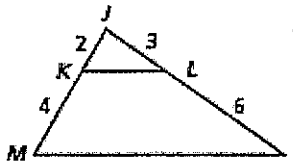
Side- Side- Side Similarity (SSS~) – If the three sides of one triangle are proportional to the three corresponding sides of another triangle, then the triangles are similar.



Side-Angle- Side Similarity (SAS~) – If two sides of one triangle are proportional to two corresponding sides of another triangle and their included angles are congruent, then the triangles are similar.



Examples: Determine if the triangles are similar. If so, tell why and write the similarity statement and similarity ratio.



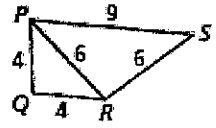
$\frac{2}{6} = \frac{3}{9}$

$\frac{1}{3} = \frac{1}{3}$ ✓

Similar : (Y) or N Why: SAS

Similarity Statement : MJN ~ KJL

Similarity Ratio : 1 : 3



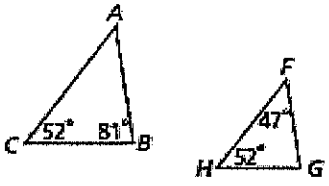
$\frac{6}{9} = \frac{4}{6}$

$\frac{2}{3} = \frac{2}{3}$ ✓

Similar : (Y) or N Why: SSS

Similarity Statement : PQR ~ PRS

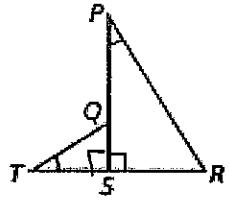
Similarity Ratio : 2 : 3



Similar : (Y) or (N) Why: AA

Similarity Statement : ABC ~ FGF

Similarity Ratio : N/A



Similar : (Y) or N Why: AA

Similarity Statement : PSR ~ TSP

Similarity Ratio : NA

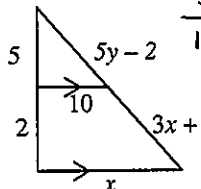
Proportional Relationships – Extra Practice

Solve for the variable in each figure.

1. $y =$ 22.4

$x =$ 14

$\frac{5}{2} = \frac{5y-2}{3(14)+2}$ → $\frac{5}{2} = \frac{5y-2}{44}$



$\frac{5}{10} = \frac{7}{x}$

$5x = 70$

$x = 14$

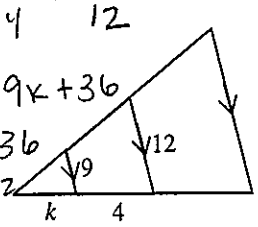
2. $k =$ 12

$\frac{k}{k+4} = \frac{9}{12}$

$12k = 9k + 36$

$3k = 36$

$k = 12$



3. SQ = x ; ST = 22 ; SP = 12 ; PR = $4x+8$

$x =$ 6

$\frac{x}{22} = \frac{12}{12+4x+8}$

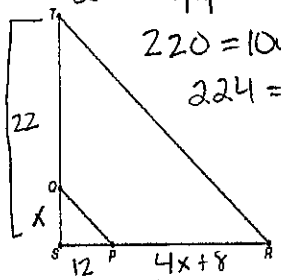
$4x + 20$

$264 = 4x^2 + 20x$

$0 = 4x^2 + 20x - 264$

$4(x^2 + 5x - 66) = 0$

$(x + 11)(x - 6)$

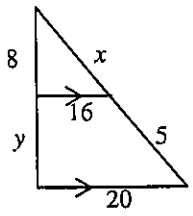


4. $y =$ 2

$x =$ 20

$220 = 10y - 4$

$224 = 10y$



$\frac{8}{8+y} = \frac{16}{20}$

$160 = 128 + 16y$

$32 = 16y$

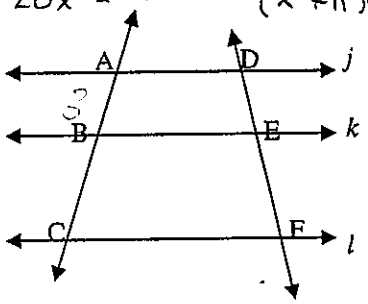
$y = 2$

$\frac{8}{2} = \frac{x}{5}$

$40 = 2x$

$x = 20$

Given: $j \parallel k \parallel l$



5. AC = 9; BC = 6; DF = 15

EF = 10

$\frac{9}{6} = \frac{15}{EF}$

$90 = 9x$

$EF = 10$

6. AB = 5y; DE = 2y; EF = 12

BC = 30

$\frac{5y}{BC} = \frac{2y}{12}$

$60y = 2y\,(BC)$

$BC = 30$

7. BC = x + 2; BA = 9; EF = x + 3; ED = 12

x = 1 ; BC = 3 ; EF = 4

$\frac{9}{x+2} = \frac{12}{x+3}$

$9x + 27 = 12x + 24$

$3 = 3x$

8. AC = 3x; BC = 16; EF = 20; FD = 4x -2

x = 8 ; AC = 24 ; FD = 30

$\frac{3x}{16} = \frac{4x-2}{20}$

$60x = 64x - 32$

$32 = 4x$

$x = 8$