

Section 6.3 Formulas

A *formula* is an equation that represents a real-life application

Example 1 Simple Interest

$$I = prt$$

I = interest

p = principal

t = time (years)

r = rate

Borrow \$4000

rate - 7%

Interest - \$840

How long did you have the loan?

$$\begin{aligned} I &= prt \\ 840 &= 4000 (.07) t \\ \frac{840}{4000(.07)} & \\ t &= 3 \text{ years} \end{aligned}$$

Example 4 page 352

Exponential Decay Formula

Carbon dating - find the age of fossils and bones

$$P = P_0 2^{\frac{-t}{5600}}$$

P_0 = original amount of carbon

P = amount present after t years

t = years

If 10 mg of carbon is present in animal bone recently excavated, then how many mg will be present after 3000 years

$$P = 10(2)^{\left(\frac{-3000}{5600}\right)} = 6.89 \text{ mg}$$

Example 5

Compound Interest Continuously

$$P = P_0 e^{kt}$$

P_0 initial investment

k = interest rate
 t = time (years)

$$e = 2.71$$

$$P_0 = 10000$$

$$(.04 \cdot 5)$$

$$= \$12214.03$$

$$- 10000$$

$$\hline 2214$$

Suppose \$10,000 is invested in a savings account at a 4% interest rate compounded continuously. What will be the balance in the account in 5 years?

$$I = 10000(.04)(5) = 2000$$

Example 6

Population Growth

$$P = P_0 e^{kt}$$

The population in CT is growing exponentially at the rate of about 3.5% per year.

In 2005, the population of CT was about 3,405,565 people. What will the population be in 2020?

$$P = 3,405,565 e^{(.035 \cdot 15)}$$
$$= 5,756,967$$

Solving for a certain variable in a formula using Algebra.

Example 7

$$\begin{aligned}
 2x + 5y - 10 &= 0 \quad \text{solve for } y \\
 -2x & \\
 5y - 10 &= -2x \\
 +10 & \\
 \frac{5}{5}y &= \frac{-2x}{5} + \frac{10}{5} \\
 y &= \frac{-2x}{5} + 2
 \end{aligned}$$

Example 8

$$\theta \cdot z = \frac{x - u}{\theta} \quad \text{solve for } x$$

$$\theta \cdot z = x - u$$

$$\theta z + u = x$$

Example 9

$$2 \cdot k = \frac{1}{2} mv^2 \quad \text{solve for } v$$

$$\frac{2k}{m} = \frac{mv^2}{m}$$

$$\sqrt{\frac{2k}{m}} = \sqrt{v^2}$$

$$v = \sqrt{\frac{2k}{m}}$$

SECTION 6.3 EXERCISES

CONCEPT/WRITING EXERCISES

1. What is a formula?
2. Explain how to evaluate a formula.
3. What are subscripts?
4. What is the simple interest formula?
5. What is an exponential equation?
 - a) In an exponential equation of the form $y = a^x$, what are the restrictions on a ?
 - b) In an exponential equation of the form $y = P_0 a^{kt}$, what does P_0 represent?

PRACTICE THE SKILLS

In Exercises 7–40, use the formula to find the value of the indicated variable for the values given. Use a calculator when one is needed. When necessary, round answers to the nearest hundredth.

7. $A = lw$; determine A when $l = 4$ and $w = 14$ (geometry).
8. $P = a + b + c$; determine P when $a = 25$, $b = 53$, and $c = 32$ (geometry).
9. $P = 2l + 2w$; determine P when $l = 12$ and $w = 16$ (geometry).
10. $F = ma$; determine m when $F = 40$ and $a = 5$ (physics).
11. $K = \frac{1}{2}mv^2$; determine m when $v = 30$ and $K = 4500$ (physics).
12. $p = i^2r$; determine r when $p = 62,500$ and $i = 5$ (electronics).
13. $S = \pi r(r + h)$; determine S when $r = 8$, $\pi = 3.14$, and $h = 2$ (geometry).
14. $B = \frac{703w}{h^2}$; determine B when $w = 130$ and $h = 67$ (body mass index).
15. $z = \frac{x - \mu}{\sigma}$; determine μ when $z = 2.5$, $x = 42.1$, and $\sigma = 2$ (statistics).
16. $S = 2hw + 2lw + 2lh$; determine l when $S = 122$, $h = 4$, and $w = 3$ (geometry).
17. $T = \frac{PV}{k}$; determine P when $T = 80$, $V = 20$, and $k = 0.5$ (physics).
18. $m = \frac{a + b + c}{3}$; determine a when $m = 70$, $b = 60$, and $c = 90$ (statistics).
19. $A = P(1 + rt)$; determine P when $A = 3600$, $r = 0.04$, and $t = 5$ (economics).
20. $m = \frac{a + b}{2}$; determine a when $m = 70$ and $b = 77$ (statistics).
21. $V = \pi r^2 h$; determine h when $V = 942$, $\pi = 3.14$, and $r = 5$ (geometry).
22. $F = \frac{9}{5}C + 32$; determine F when $C = 7$ (temperature conversion).
23. $C = \frac{5}{9}(F - 32)$; determine C when $F = 77$ (temperature conversion).
24. $K = \frac{F - 32}{1.8} + 273.15$; determine K when $F = 100$ (chemistry).
25. $m = \frac{y_2 - y_1}{x_2 - x_1}$; determine m when $y_2 = 8$, $y_1 = -4$, $x_2 = -3$, and $x_1 = -5$ (mathematics).
26. $z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$; determine z when $\bar{x} = 66$, $\mu = 60$, $\sigma = 15$, and $n = 25$ (statistics).
27. $S = R - rR$; determine R when $S = 186$ and $r = 0.07$ (for determining sale price when an item is discounted).
28. $S = C + rC$; determine C when $S = 115$ and $r = 0.15$ (for determining selling price when an item is marked up).
29. $E = a_1 p_1 + a_2 p_2 + a_3 p_3$; determine E when $a_1 = 5$, $p_1 = 0.2$, $a_2 = 7$, $p_2 = 0.6$, $a_3 = 10$, and $p_3 = 0.2$ (probability).
30. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$; determine x when $a = 2$, $b = -5$, and $c = -12$ (mathematics).

Section 6.3 Solving For Variables in Formulas

55. $V = \frac{1}{3} Bh$ for B

$$\frac{3V}{h} = B$$

56. $V = \pi r^2 h$ for h

$$\frac{V}{\pi r^2} = h$$

$$57. \frac{C}{2\pi} = \frac{2\pi r}{2\pi} \text{ for } r$$

$$r = \frac{C}{2\pi}$$

$$58. y = mx + b \text{ for } m$$

$$y - b = mx$$

$$m = \frac{y - b}{x}$$

$$59. y = mx + b \text{ for } b$$

$$y - mx = b$$

$$60. V = \frac{3}{4} \cdot \frac{1}{3} \pi r^2 h \text{ for } h$$

$$\frac{3V}{\pi r^2} = \frac{\pi r^2 h}{\pi r^2} \quad h = \frac{3V}{\pi r^2}$$

$$61. P = 2l + 2w \quad \text{for } w \quad 62. A = \frac{d_1 d_2}{2} \quad \text{for } d_2$$

$$\frac{P}{2} - \frac{2l}{2} = \frac{2w}{2}$$

$$\frac{P}{2} - l = w$$

$$\frac{2A}{d_1} = \frac{d_1 \cancel{d_2}}{\cancel{d_1}}$$

$$d_2 = \frac{2A}{d_1}$$

$$63. A = \frac{a + b + c}{3} \quad \text{for } c \quad 64. \frac{w_1}{w_2} = \frac{f_2 - f}{f - f_1} \quad \text{for } w_1$$

$$3A = a + b + c$$

$$-a - b$$

$$3A - a - b = c$$

$$w_1 = \frac{(f_2 - f) \cdot w_2}{f - f_1}$$

$$65. P = \frac{KT}{V} \quad \text{for } T$$

$$66. \frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \quad \text{for } V_2$$

$$67. F = \frac{9}{5}C + 32 \quad \text{for } C \quad 68. C = \frac{5}{9}(F - 32) \quad \text{for } F$$