Section 6.5 Variation

Direct Variation
The values of two related variables increase together or decrease together

Distance formula

$d=rt$
↑ constant

The distance varies directly as the time you drive.

Example:
rate is 40 miles per hour

2 hours -> 80 miles
3 hours -> 120 miles

the more time the more miles traveled

$40 = k$
↖ constant of proportionality

Direct Variation: multiplication

$y = kx$
↑ constant of proportionality

EXAMPLE 1 *Direct Variation in Physics*

The length that a spring will stretch, S, varies directly with the forc F
attached to the spring. Write the equation for the length that a sprir
if the constant of proportionality is 0.07.

$S = kF$

$S = .07F$

EXAMPLE 2 *Direct Variation in Medicine*

The recommended dosage, d, of the antibiotic drug vancomycin is direct vari
tional to a person's weight, w.

a) Write this variation as an equation.

b) Find the recommended dosage, in milligrams, for Doug Kulzer, 192 lb. Assume the constant of proportionality for the dosage is 18.

a) $d = kw$

b) $d = 18(192) = 3456 \text{ mg}$

EXAMPLE ❸ *Finding the Constant of Proportionality*

Suppose w varies directly as the square of y. If w is 60 when y is 20, find the constant of proportionality.

$w = ky^2$

$\frac{60}{20^2} = \frac{k20^2}{20^2}$

$K = .15$

EXAMPLE ❹ *Using the Constant of Proportionality*

The area, a, of a picture projected on a movie screen varies directly as the square of the distance, d, from the projector to the screen. If a projector at a distance of 25 feet projects a picture with an area of 100 square feet, what is the area of the projected picture when the projector is at a distance of 40 feet?

$a = kd^2$

$100 = k(25)^2$

$K = .16$

$a = .16(40)^2$

$a = 256\ ft^2$

Inverse variation

When two quantities vary inversely, as one quantity increases the other quantity decrease, and vice versa.

d=rt solve for t

$t = \frac{d}{r}$

d is the constant

d = 100 miles

If r = 50, then t = _____ $\frac{100}{50} = 2$ hr

If r = 25, then t = _____ $\frac{100}{25} = 4$ hr

If r = 10, then t = _____ $\frac{100}{10} = 10$ hr

Inverse variation:

$y = \frac{k}{x}$ ← constant

EXAMPLE 5 *Inverse Variation in Astronomy*

The velocity, v, of a meteor approaching Earth varies inversely as the square root of its distance from the center of Earth. Assuming the velocity is 2 miles per second at a distance, d, of 6400 miles from the center of Earth, determine the equation that expresses the relationship between the velocity of a meteor and its distance from the center of Earth.

$v = \frac{k}{\sqrt{d}}$

$2 = \frac{k}{\sqrt{6400}}$

$k = 160$

$v = \frac{160}{\sqrt{d}}$

$v = \frac{160}{\sqrt{4000}}$

EXAMPLE 6 *Using the Constant of Proportionality*

Suppose y varies inversely as x. If $y = 8$ when $x = 15$, find y when $x = 18$.

$y = \frac{k}{x}$

$15 \cdot 8 = \frac{k}{15} \cdot 15$

$120 = k$

$y = \frac{120}{18}$

$y = 6.67$

5. The distance between two cities on a map and the actual distance between the two cities

6. The time required to fill a pool with a hose and the volume of water coming from the hose

7. The time required to boil water on a burner and the temperature of the burner

8. A person's salary and the amount of money withheld from his or her salary for federal income taxes

9. The interest earned on an investment and the interest rate

10. The volume of a balloon and its radius

$100\% \times -20\% x = 33$

$.80x = \frac{33}{.80}$

11. A person's speed and the time needed for the person to complete the race

12. The time required to cool a room and the temperature of the room

13. The number of workers hired to install a fence and the time required to install the fence

14. The number of calories in a slice of pizza and the size of the slice

15. The time required to defrost a frozen hamburger in a room and the temperature of the room

16. On Earth, the weight and mass of an object

17. The number of people in line at a bank and the amount of time required for the last person to reach the teller

18. The number of books that can be placed upright on a shelf 3 ft long and the width of the books

19. The amount of fertilizer needed to fertilize a lawn and the area of the lawn

20. The percent of light that filters through water and the depth of the water

Joint Variation

One quantity may vary directly as a ***product*** of two or more other quantities.

The general form of a joint variation, where y varies directly as x and z, is

$y = kxz$

where k is the constant of proportionality.

EXAMPLE 7 *Joint Variation in Geometry*

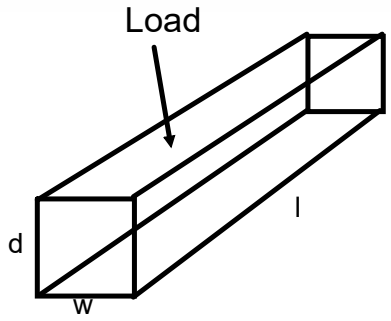
The area, A, of a triangle varies jointly as its base, b, and height, h. If the area of a triangle is 48 $in.^2$ when its base is 12 in. and its height is 8 in., find the area of a triangle whose base is 15 in. and whose height is 20 in.

SUMMARY OF VARIATIONS

Direct	Inverse	Joint
$y = kx$	$y = \frac{k}{x}$	$y = kxz$

EXAMPLE 8 *Combined Variation in Engineering*

The load, L, that a horizontal beam can safely support varies jointly as width, w, and the square of the depth, d, and inversely as the length, l. Express L, in terms w, d, l, and the constant of proportionality, k.



28. M varies directly as J and inversely as C. I
when $J = 10$, $C = 20$, and $k = 8$.

EXAMPLE 9 *Hot Dog Price, Combined Variation*

The owners of Henrietta Hots find their weekly sales of hot dogs, S, varies directly with their advertising budget, A, and inversely with their hot dog price P, when their advertising budget is \$600 and the price of a hot dog is \$1.50, they sell 5600 hot dogs a week.

EXAMPLE 10 *Combined Variation*

A varies jointly as B and C and inversely as the square of D. If $A = 1$ when $B = 9$, $C = 4$, and $D = 6$, find A when $B = 8$, $C = 12$, and $D = 5$.

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#41, 42-48 even

Exercises 41–49, (a) write the variation and (b) deter-ıe the quantity indicated.

Property Tax The property tax, t, on a home is directly proportional to the assessed value, v, of the home. If the property tax on a home with an assessed value of \$140,000 is \$2100, what is the property tax on a home with an assessed value of \$180,000?

Resistance The resistance, R, of a wire varies directly as its length, L. If the resistance of a 30 ft length of wire is 0.24 ohm, determine the resistance of a 40 ft length of wire.

Speaker Loudness The loudness of a stereo speaker, l, measured in decibels (dB), is inversely proportional to the square of the distance, d, of the listener from the speaker. If the loudness is 20 dB when the listener is 6 ft from the speaker, what is the loudness when the listener is 3 ft from the speaker?

Melting an Ice Cube The time, t, for an ice cube to melt is inversely proportional to the temperature, T, of the water in which the ice cube is placed. If it takes an ice cube 2 minutes to melt in 75°F water, how long will it take an ice cube of the same size to melt in 80°F water?

45. ***Video Rentals*** The number of weekly videotape rentals, R, at Busterblock Video varies directly with the advertising budget, A, and inversely with the daily rental price, P. When the video store's advertising budget is \$600 and the rental price is \$3 per day, it rents 4800 tapes per week. How many tapes would it rent per week if the store increased its advertising budget to \$700 and raised its rental price to \$3.50?

46. ***Stopping Distance of a Car*** The stopping distance, d, of a car after the brakes are applied varies directly as the square of the speed, s, of the car. If a car traveling at a speed of 40 mph can stop in 80 ft, what is the stopping distance of a car traveling at 65 mph?

47. ***Guitar Strings*** The number of vibrations per second, v, of a guitar string varies directly as the square root of the tension, t, and inversely as the length of the string, l. If the number of vibrations per second is 5 when the tension is 225 kg and the length of the string is 0.60 m, determine the number of vibrations per second when the tension is 196 kg and the length of the string is 0.70 m.

48. ***Electrical Resistance*** The electrical resistance of a wire, R, varies directly as its length, L, and inversely as its cross-sectional area, A. If the resistance of a wire is 0.2 ohm when the length is 200 ft and its cross-sectional area is 0.05 $in.^2$, what is the resistance of a wire whose length is 5000 ft with a cross-sectional area of 0.01 $in.^2$?