Section 4.4 Polynomial Functions

1 Polynomial Functions A **polynomial in one variable** is an expression of the form $a_nx^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0$, where $a_n \neq 0$, a_{n-1}, a_2, a_1, and a_0 are real numbers, and n is a nonnegative integer.

The polynomial is written in standard form when the values of the exponents are in descending order. The degree of the polynomial is the value of the greatest exponent. The coefficient of the first term of a polynomial in standard form is called the **leading coefficient**.

Polynomial	Expression	Degree	Leading Coefficient
Constant	12	0	12
Linear	$4x - 9$	1	4
Quadratic	$5x^2 - 6x - 9$	2	5
Cubic	$8x^3 + 12x^2 - 3x + 1$	3	8
General	$a_nx^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0$	n	a_n

Example 1 Degrees and Leading Coefficients

State the degree and leading coefficient of each polynomial in one variable. If it is not a polynomial in one variable, explain why.

a. $8x^5 - 4x^3 + 2x^2 - x - 3$

yes 5^{th} 8

b. $12x^2 - 3xy + 8x$

NO 2^{nd} 12

c. $3x^4 + 6x^3 - 4x^8 + 2x$

yes 8^{th} -4

1A. $5x^3 - 4x^2 - 8x + \frac{4}{x}$

NO 3^{rd} 5

1B. $5x^6 - 3x^4 + 12x^3 - 14$

yes 6^{th} 5

1C. $8x^4 - 2x^3 - x^6 + 3$

yes 6^{th} -1

The volume of air in the lungs during a 5-second respiratory cycle can be modeled by $v(t) = -0.037t^3 + 0.152t^2 + 0.173t$, where v is the volume in liters and t is the time in seconds. This model is an example of a polynomial function.

Real-World Example 2 Evaluate a Polynomial Function

RESPIRATION **Refer to the beginning of the lesson. Find the volume of air in the lungs 2 seconds into the respiratory cycle.**

By substituting 2 into the function we can find $v(2)$, the volume of air in the lungs 2 seconds into the respiratory cycle.

$$v(2) = -.037(2)^3 + .152(2)^2 + .173(2) = .658$$

2. Find the volume of air in the lungs 4 seconds into the respiratory cycle.

$$v(4) = .756$$

Find $p(-6)$ and $p(3)$ for each function.

23. $p(x) = x^4 - 2x^2 + 3$

$p(-6) = (-6)^4 - 2(-6)^2 + 3 = 1227$

$p(3) = 3^4 - 2(3)^2 + 3 = 66$

24. $p(x) = -3x^3 - 2x^2 + 4x - 6$

25. $p(x) = 2x^3 + 6x^2 - 10x$

26. $p(x) = x^4 - 4x^3 + 3x^2 - 5x + 24$

$p(-6) = 2332$

$p(3) = 9$

If $c(x) = 2x^2 - 4x + 3$ and $d(x) = -x^3 + x + 1$, find each value.

29. $c(3a) = 2(3a)^2 - 4(3a) + 3$

$= 2(9a^2) - 12a + 3$

$= 18a^2 - 12a + 3$

31. $c(b^2)$

$2(b^2)^2 - 4(b^2) + 3$

$2b^4 - 4b^2 + 3$

$d(4a^2) = -(4a^2)^3 + 4a^2 + 1$

$-64a^6 + 4a^2 + 1$

If $p(x) = 3x^2 - 4$ and $r(x) = 2x^2 - 5x + 1$, find each value.

$p(8a)$ $r(a^2)$ $-5r(2a)$

$r(x+2)$

$2(x+2)^2 - 5(x+2) + 1$

$2(x+2)(x+2) - 5x - 10 + 1$

$2(x^2 + 2x + 2x + 4)$

$4x$

$2x^2 + 8x + 8 - 5x - 9$

$2x^2 + 3x - 1$

$p(x^2-1)$

$3(x^2-1)^2 - 4$

$3(x^2-1)(x^2-1) - 4$

$3(x^4 - x^2 - x^2 + 1) - 4$

$3x^4 - 3x^2 - 3x^2 + 3 - 4$

$3x^4 - 6x^2 - 1$

If $p(x) = 3x^2 - 4$ and $r(x) = 2x^2 - 5x + 1$, find each value.

$5p(x+2)$

$= 3(x+2)(x+2) - 4$

$= 3(x^2 + 2x + 2x + 4) - 4$

$(3x^2 + 6x + 6x + 12) - 4$

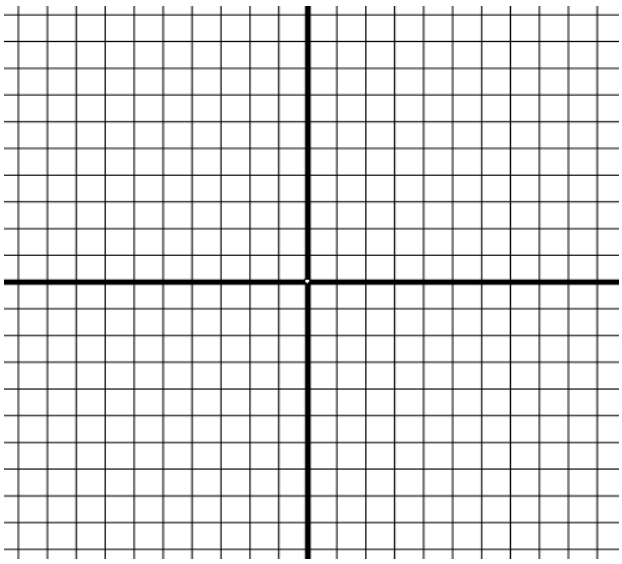
$5(3x^2 + 12x + 8)$

$15x^2 + 60x + 40$

Use your graphing calculator to graph the following polynomial functions. Find each zero and each max/min.

$f(x) = x^3 - x^2 - 6x$

rough sketch



zeros:

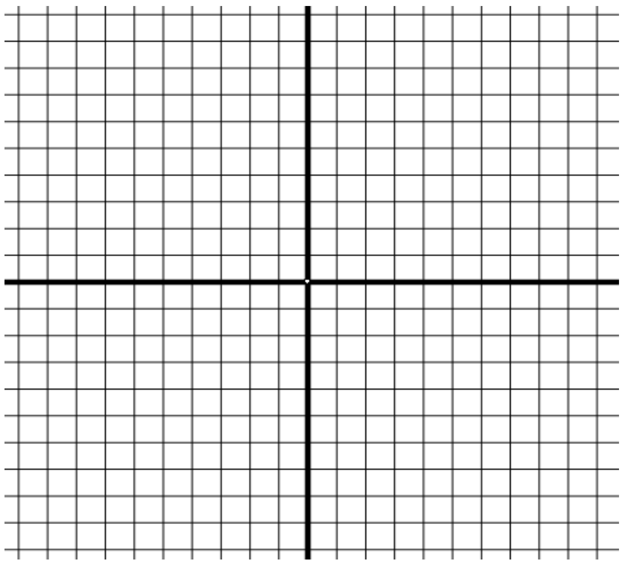
Max coordinates:

Min coordinates:

Use your graphing calculator to graph the following polynomial functions. Find each zero and each max/min.

rough sketch

$f(x) = x^4 - 8x^3 + 19x^2 - 12x$



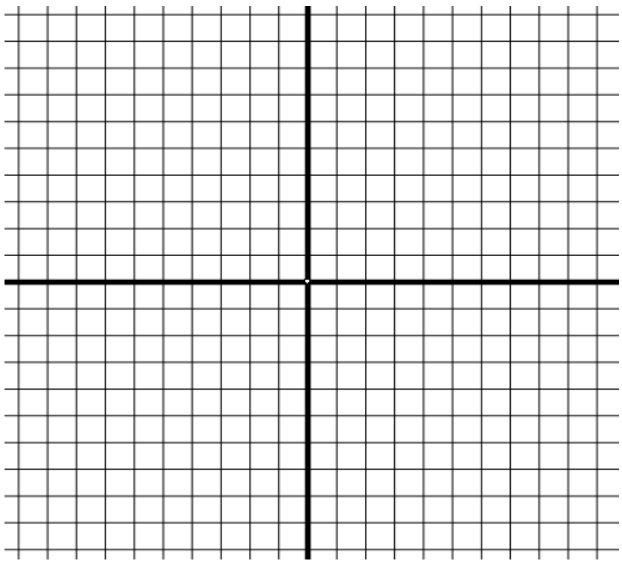
zeros:

Max coordinates: Min coordinates:

Use your graphing calculator to graph the following polynomial functions. Find each zero and each max/min.

rough sketch

$$f\left(x\right)=x^4+2x^3-3x^2$$



zeros:

Max coordinates:

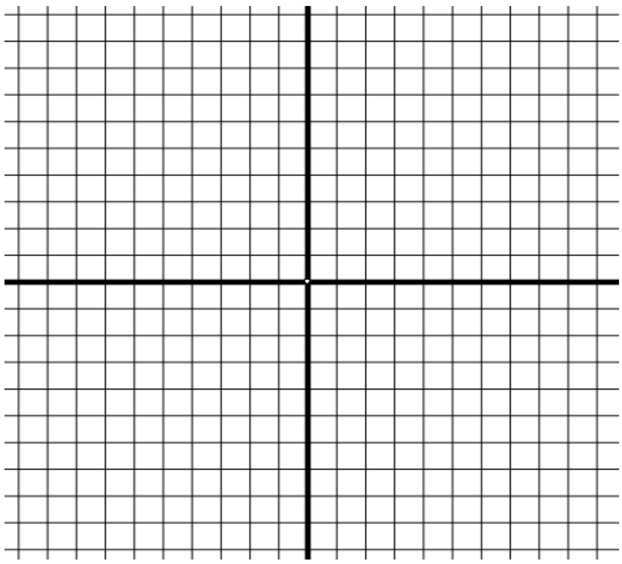
Min coordinates:

Use your graphing calculator to graph the following polynomial functions. Find each zero and each max/min.

rough sketch

$$f(x) = x^2 + 3x - 4$$

zeros:



Max coordinates:

Min coordinates:

Find the minimum, maximum and zeros of each function below.

$f(x) = 2x^4 + 4x^3 - 3x$

$g(x) = (x-2)(x+3)(x)$

$f(x) = x^5 - 3x^3 + 2x - 1$

$f(x) = x^4 - 2x^3 + x^2 - x + 2$

$f(x) = x^5 + 2x^4 + 4x^3 + 2x^2 - 3x - 1$

$g(x) = 3x^4 + 2x - 4$