Section 5.2

Composition of Functions

1 Perform Compositions of Functions

Perform Compositions of Functions You have already combined functions with arithmetic operations. Another method used to combine functions is a composition of functions. In a **composition of functions**, the results of one function are used to evaluate a second function.

$f(2)$

$f(x) = x + 6$ $g(x) = x^2 - 3x + 1$

$f(g(x))$

$f(x^2 - 3x + 1)$

$(x^2 - 3x + 1) + 6$

$x^2 - 3x + 7$

$g(f(x))$

$g(x+6)$

$(x+6)(x+6) - 3(x+6) + 1$

$x^2 + 6x + 6x + 36 - 3x - 18 + 1$

$x^2 + 9x + 19$

$f(x) = 2x - 1$ $(2x-1)^2$ $g(x) = x^2 + 2x - 2$

Find

$(g \circ f)(x)$

$g(f(x))$

$g(2x-1) = (2x-1)(2x-1) + 2(2x-1) - 2$

$4x^2 - 2x - 2x + 1 + 4x - 2 - 2$

$4x^2 - 3$

$(f \circ g)(x)$

$f(g(x))$

$f(x^2 + 2x - 2)$

$= 2(x^2 + 2x - 2) - 1$

$2x^2 + 4x - 4 - 1$

$\boxed{2x^2 + 4x - 5}$

$f(x) = 2x - 3$ $\qquad$ $g(x) = x^2 - 4x$

Find

$(g \circ f)(x)$

$g(2x-3)$

$= (2x-3)(2x-3) - 4(2x-3)$

$4x^2 - 6x - 6x + 9 - 8x + 12$

$4x^2 - 20x + 21$

$(f \circ g)(x)$

$f(x^2 - 4x)$

$= 2(x^2 - 4x) - 3$

$= 2x^2 - 8x - 3$

$f(x) = x - 5$ $\qquad$ $g(x) = x^2 + x - 1$

Find

$f(g(x))$

$g(f(x))$

$g(x-5)$

$= (x-5)(x-5) + x - 5 - 1$

$x^2 - 5x - 5x + 25 + x - 5 - 1$

$x^2 - 9x + 19$

Given $f(x) = x^2 - 4$ and $g(x) = 2x + 1$, find each value.

a. $[f \circ g](3)$

$f(g(3))$ $f(2(3)+1)$ $f(7) = 7^2 - 4$
$= 49 - 5 = 45$

$3 \to 45$

b. $[g \circ f](3)$ $f(3) = 3^2 - 4 = 5$ $g(5) = 2(5) + 1 = 11$

$3, 11$

Given $f(x) = 3x - 6$ and $g(x) = x^3 + 1$, find each value.

1A. $[f \circ g](5)$ **1B.** $[g \circ f](5)$

The composition of two functions may not exist. Given two functions f and g, $[f \circ g](x)$ is defined only if the range of $g(x)$ is a subset of the domain of f. Likewise, $[g \circ f](x)$ is defined only if the range of $f(x)$ is a subset of the domain of g.

Example 2 Perform Compositions of Functions

For each pair of functions, find $[f \circ g](x)$ and $[g \circ f](x)$, if they exist. State the domain and range for each composed function.

a. $f = \{(1, 8), (0, 13), (15, 11), (14, 9)\}, g = \{(8, 15), (5, 1), (10, 14), (9, 0)\}$

To find $f \circ g$, evaluate $g(x)$ first. Then use the range to evaluate $f(x)$.

$f[g(8)] = f(15)$ or 11 $g(8) = 15$

$f[g(5)] = f(1)$ or 8 $g(5) = 1$

$f[g(10)] = f(14)$ or 9 $g(10) = 14$

$f[g(9)] = f(0)$ or 13 $g(9) = 0$

$f \circ g = \{(8, 11), (5, 8), (10, 9), (9, 13)\}$

D = {5, 8, 9, 10}, R = {8, 9, 11, 13}

$f = \{(1, 8), (0, 13), (15, 11), (14, 9)\}, g = \{(8, 15), (5, 1), (10, 14), (9, 0)\}$

To find $g \circ f$, evaluate $f(x)$ first. Then use the range to evaluate $g(x)$.

$g[f(1)] = g(8)$ or 15 $f(1) = 8$ (1, 15)

$g[f(0)] = g(13)$ $g(13)$ is undefined.

$g[f(15)] = g(11)$ $g(11)$ is undefined.

$g[f(14)] = g(9)$ or 0 $f(14) = 0$ (14, 0)

Because 11 and 13 are not in the domain of g, $g \circ f$ is undefined for $x = 0$ and $x = 15$. However, $g[f(1)] = 15$ and $g[f(14)] = 0$.

So, $g \circ f = \{(1, 15), (14, 0)\}$.

D = {1, 14}, R = {0, 15}

For each pair of functions, find $[f \circ g](x)$ and $[g \circ f](x)$, if they exist. State the domain and range for each combined function.

a. $f = \{(2, 6), (9, 4), (7, 7), (0, -1)\}$ and $g = \{(7, 0), (-1, 7), (4, 9), (8, 2)\}$

b. $f(x) = 2x - 5$, $g(x) = 4x$

$[f \circ g](x) = f[g(x)]$ $[g \circ f](x) = g[f(x)]$

For $[f \circ g](x)$, D = {all real numbers} and R = {all real numbers}, and for $[g \circ f](x)$, D = {all real numbers} and R = {all real numbers}.

For each pair of functions, find $[f \circ g](x)$ and $[g \circ f](x)$, if they exist. State the domain and range for each composed function.

2A. $f(x) = \{(3, -2), (-1, -5), (4, 7), (10, 8)\}$, $g(x) = \{(4, 3), (2, -1), (9, 4), (3, 10)\}$

For each pair of functions, find $[f \circ g](x)$ and $[g \circ f](x)$, if they exist. State the domain and range for each composed function.

2B. $f(x) = x^2 + 2$ and $g(x) = x - 6$

Given $f(x) = x^2 + 3$ and $g(x) = 2x - 10$, find each value.

1. $[f \circ g](4)$
2. $[g \circ f](4)$
3. $[f \circ g](0)$
4. $[g \circ f](0)$
5. $[f \circ g](-1)$
6. $[g \circ f](-1)$

For each pair of functions, find $f \circ g$ and $g \circ f$, if they exist. State the domain and range for each composed function.

10. $f = \{(2, 5), (6, 10), (12, 9), (7, 6)\}$
$g = \{(9, 11), (6, 15), (10, 13), (5, 8)\}$

For each pair of functions, find $f \circ g$ and $g \circ f$, if they exist. State the domain and range for each composed function.

11. $f = \{(-5, 4), (14, 8), (12, 1), (0, -3)\}$
$g = \{(-2, -4), (-3, 2), (-1, 4), (5, -6)\}$

For each pair of functions, find $f \circ g$ and $g \circ f$, if they exist. State the domain and range for each composed function.

12. $f(x) = -3x$
$g(x) = 5x - 6$