Quadratic Velocity Applications

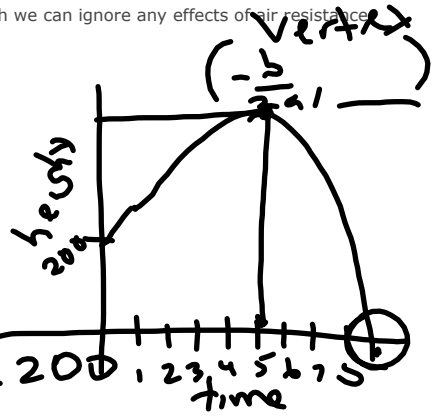
Let's solve the example of a quadratic equation involving maximums and minimums for projectile motion given above: A ball is thrown directly upward from an initial height of 200 feet with an initial velocity of 96 feet per second. After how many seconds will the ball reach its maximum height? And, what is the maximum height?
To analyze our problems, we will be using a formula for a freely falling body in which we can ignore any effects of air resistance.

- $s(t)$ represents the projectile's instantaneous height at any time t
- v_o represents initial velocity
- s_o represents the initial height from which the projectile is released
- t represents time in seconds after the projectile is released

$$\frac{-96}{2(-16)} = 3$$

$$s(t) = -16t^2 + v_0 t + s_0$$

$$S(t) = -16t^2 + 96t + 200$$



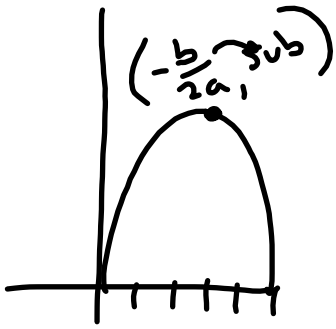
1. Some fireworks are fired vertically into the air from the ground at an initial velocity of 80 feet per second. Find the highest point reached by the projectile just as it explodes.

$$h(t) = -16t^2 + 80t + 0$$

$$\text{time} = \frac{-80}{2(-16)} = \frac{-80}{-32} = 2.5 \text{ sec}$$

$$h(2.5) = -16(2.5)^2 + 80(2.5)$$

$$= 100 \text{ ft}$$



1. time to the top is $\frac{-b}{2a}$ (axis of symmetry)

2. Find maximum height
   - take axis of symmetry and plug it in for t

$$s(t) = -16t^2 + 96t + 200$$
$$= -16(3^2) + 96(3) + 200$$
$$= 344 ft$$

3) How long before object hits ground?

Quadratic Formula $x =$

$$x = \frac{-96 \pm \sqrt{96^2 - 4(-16)(200)}}{2(-16)} = \frac{-96 \pm 148.4}{-32}$$

$$= \frac{-96 - 148.4}{-32}$$

$$x = 7.6 \text{ sec.}$$

1. Some fireworks are fired vertically into the air from the ground at an initial velocity of 80 feet per second. Find the highest point reached by the projectile just as it explodes.

$$s(t) = -16t^2 + v_0 t + s_0$$

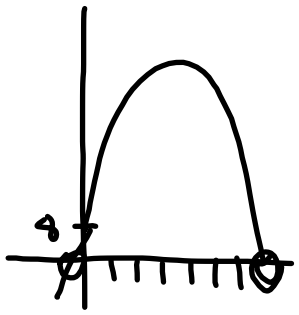
A ball is thrown vertically upward with an initial velocity of 48 feet per second. If the ball started from a height of 8 feet off the ground, determine the time it will take for the ball to hit the ground.

$h(t) = -16t^2 + 48t + 8$

$x = \dfrac{-48 \pm \sqrt{48^2 - 4(-16)(8)}}{2(-16)}$

$x = \dfrac{-48 \pm 53.1}{-32}$

$\dfrac{-48 - 53.1}{-32} = 3.2 \text{ sec}$



$\dfrac{-48 + 53.1}{-32} = -.16$

$$s(t) = -16t^2 + v_0 t + s_0$$

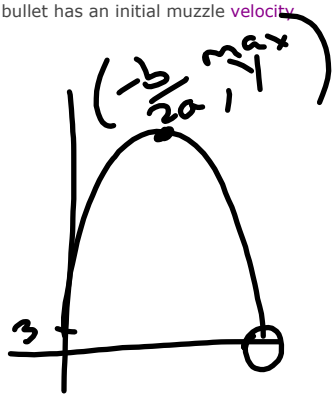
1. A pistol is accidentally discharged vertically upward at a height of 3 feet above the ground. If the bullet has an initial muzzle velocity of 200 feet per second, what maximum height will it reach before it starts to fall to the ground?

$h(t) = -16t^2 + 200t + 3$

$\text{time} = \frac{-200}{-32} = 6.25 \text{ sec}$

$= -16(6.25)^2 + 200(6.25) + 3$

$= 628 \text{ ft}$



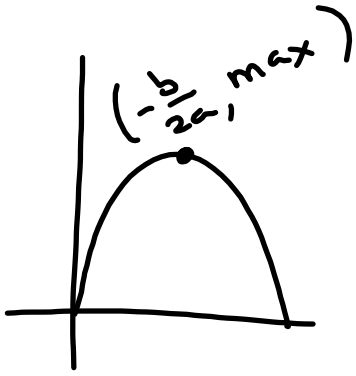
1. An over zealous golfer hits a flop shot with a sand wedge to get out of the corner of a sand trip with an initial velocity of 45 feet per second. What is the maximum height that the golf ball will reach?

$h(t) = -16t^2 + 45t + 0$

$x = \frac{-45}{-32} = 1.4 \text{ sec}$

$x = -16(1.4)^2 + 45(1.4)$

$= 31.6$



Quadratic word problems

An object is launched at 19.6 meters per second (m/s) from a 58.8-meter tall platform. The equation for the object's height s at time t seconds after launch is $s(t) = -4.9t^2 + 19.6t + 58.8$, where s is in meters. When does the object strike the ground?

2. An object in launched directly upward at 64 feet per second (ft/s) from a platform 80 feet high. The equation of the object's height, h, at time, t, seconds after launch is $h(t) = -16t^2 + 64t + 80$. What will be the object's maximum height? When will it attain this height?

When a flare is fired upward at 58.8 m/s, its height, h meters, is given by the equation h = -4.9t^2 + 58.8t, where t seconds is the time since firing.

a) Determine the maximum height of the flare and the time it takes to reach this height.

1. An object is launched from ground level directly upward at 39.2 m/s. The equation of the object's height, h, at time, t, seconds after launch is $h(t) = -4.9t^2 + 39.9t$.

How many seconds will it take for the object to hit the ground?