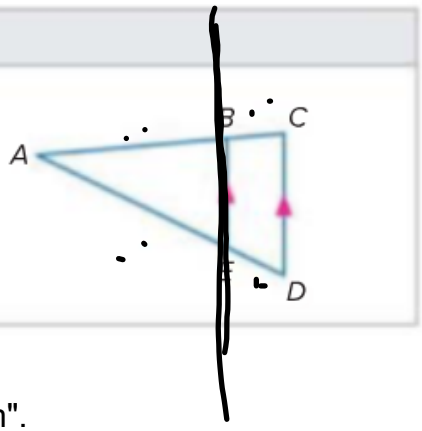
Parallel Lines and Proportional Parts

1 Proportional Parts Within Triangles When a triangle contains a line that is parallel to one of its sides, the two triangles formed can be proved similar using the Angle-Angle Similarity Postulate. Because the triangles are similar, their sides are proportional.

Theorem 7.5 Triangle Proportionality Theorem

If a line is parallel to one side of a triangle and intersects the other two sides, then it divides the sides into segments of proportional lengths.

Example If $\overline{BE} \parallel \overline{CD}$, then $\frac{AB}{BC} = \frac{AE}{ED}$.



This also known as "The Side Splitter Theorem".

Example 1 Find the Length of a Side

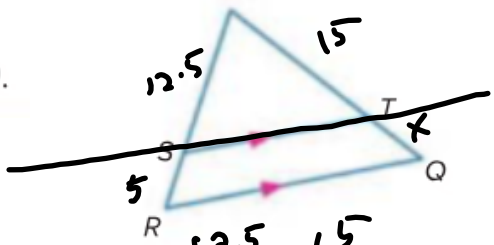
In $\triangle PQR$, $\overline{ST} \parallel \overline{RQ}$. If $PT = 7.5$, $TQ = 3$, and $SR = 2.5$, find PS.

$\frac{x}{2.5} = \frac{7.5}{3}$

$x = 6.25$

Guided Practice

1. If $PS = 12.5$, $SR = 5$, and $PT = 15$, find TQ.

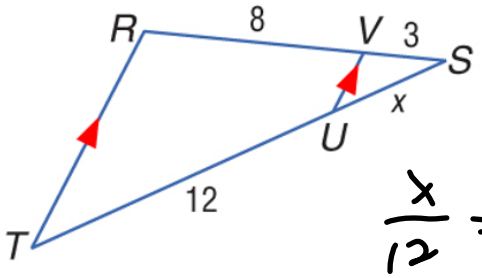


$\frac{12.5}{5} = \frac{15}{x}$

$12.5x = 75$

$x = 6$

In $\triangle RST$, $\overline{RT} \parallel \overline{VU}$, $SV = 3$, $VR = 8$, and $UT = 12$. Find SU.



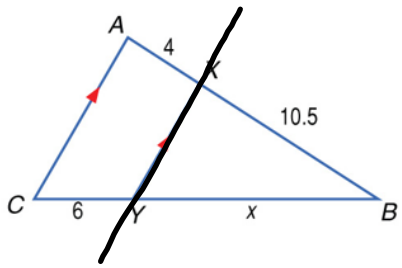
$$\frac{x}{12} = \frac{3}{8}$$

$$8x = 36$$

$$x = \frac{9}{2}$$

In $\triangle ABC$, $\overline{AC} \parallel \overline{XY}$, $AX = 4$, $XB = 10.5$, and $CY = 6$.

Find BY.



$$\frac{4}{10.5} = \frac{6}{x}$$

$$4x = 63$$

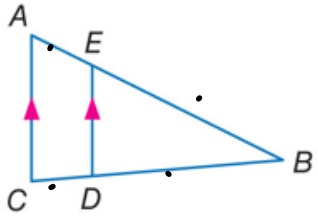
$$x = 18.75$$

15.75

Theorem 7.6 Converse of Triangle Proportionality Theorem

If a line intersects two sides of a triangle and separates the sides into proportional corresponding segments, then the line is parallel to the third side of the triangle.

Example If $\frac{AE}{EB} = \frac{CD}{DB}$, then $\overline{AC} \parallel \overline{ED}$.

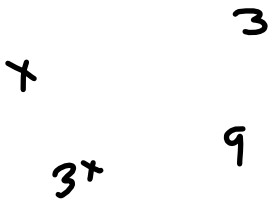


Example 2 Determine if Lines Are Parallel

In $\triangle DEF$, $EH = 3$, $HF = 9$, and DG is one-third the length of $\overline{GF}$. Is $\overline{DE} \parallel \overline{GH}$?

YES $\frac{1x}{3x} \stackrel{?}{=} \frac{3}{9}$

$\frac{1}{3} = \frac{3}{9}$



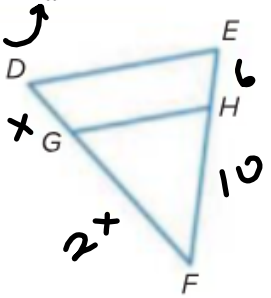
Guided Practice

2. DG is half the length of $\overline{GF}$, $EH = 6$, and $HF = 10$. Is $\overline{DE} \parallel \overline{GH}$?

$\frac{x}{2x} \neq \frac{6}{10}$

$10x \neq 12x$

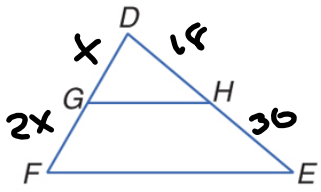
NO



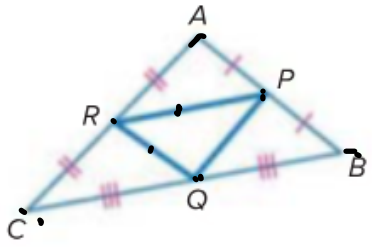
In $\triangle DEF$, $DH = 18$, $HE = 36$, and $DG = \frac{1}{2}GF$.

Determine whether $\overline{GH} \parallel \overline{FE}$. Explain.

$\frac{1x}{2x} = \frac{18}{36}$ yes



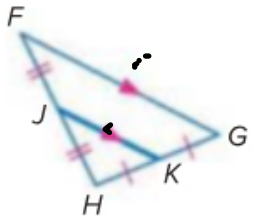
A **midsegment of a triangle** is a segment with endpoints that are the midpoints of two sides of the triangle. Every triangle has three midsegments. The midsegments of $\triangle ABC$ are $\overline{RP}$, $\overline{PQ}$, and $\overline{RQ}$.



Theorem 7.7 Triangle Midsegment Theorem

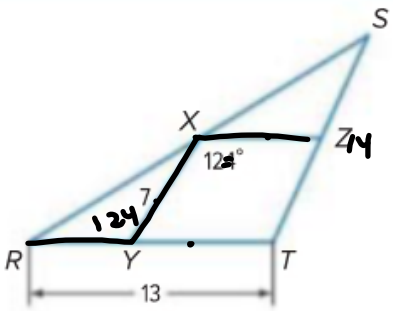
A midsegment of a triangle is parallel to one side of the triangle, and its length is one half the length of that side.

Example If J and K are midpoints of $\overline{FH}$ and $\overline{HG}$, respectively, then $\overline{JK} \parallel \overline{FG}$ and $JK = \frac{1}{2}FG$.



In the figure, $\overline{XY}$ and $\overline{XZ}$ are midsegments of $\triangle RST$. Find each measure.

a. XZ 13 ÷ 2 = 6.5



b. ST 14

c. $m\angle RYX$

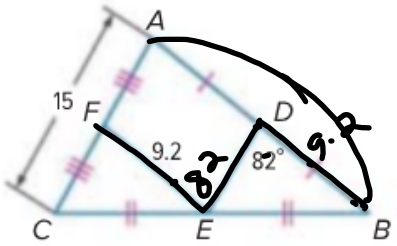
Guided Practice

Find each measure.

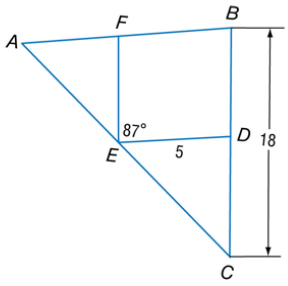
3A. DE 7.5

3B. DB 9.2

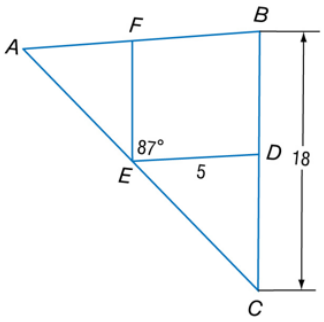
3C. $m\angle FED$ 82



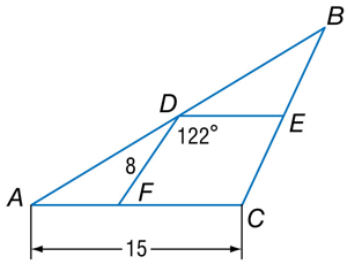
A. In the figure, $\overline{DE}$ and $\overline{EF}$ are midsegments of ΔABC. Find AB.



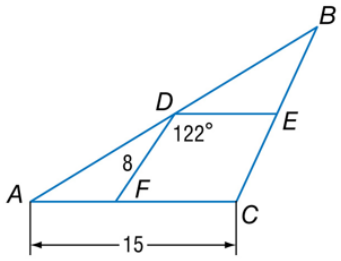
B. In the figure, $\overline{DE}$ and $\overline{EF}$ are midsegments of ΔABC. Find FE.



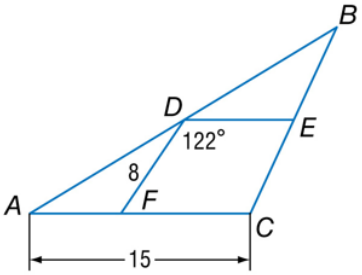
A. In the figure, $\overline{DE}$ and $\overline{DF}$ are midsegments of ΔABC. Find BC.



B. In the figure, $\overline{DE}$ and $\overline{DF}$ are midsegments of ΔABC. Find DE.

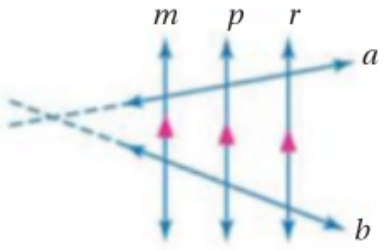


C. In the figure, $\overline{DE}$ and $\overline{DF}$ are midsegments of ΔABC. Find $m\angle AFD$.



2 Proportional Parts with Parallel Lines

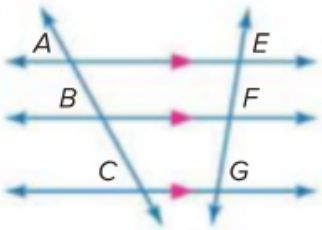
Another special case of the Triangle Proportionality Theorem involves three or more parallel lines cut by two transversals. Notice that if transversals a and b are extended, they form triangles with the parallel lines.



Corollary 7.1 Proportional Parts of Parallel Lines

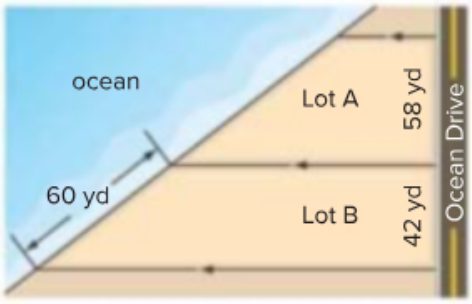
If three or more parallel lines intersect two transversals, then they cut off the transversals proportionally.

Example If $\overline{AE} \parallel \overline{BF} \parallel \overline{CG}$, then $\frac{AB}{BC} = \frac{EF}{FG}$.

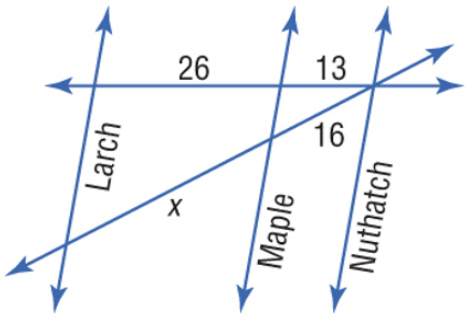


Guided Practice

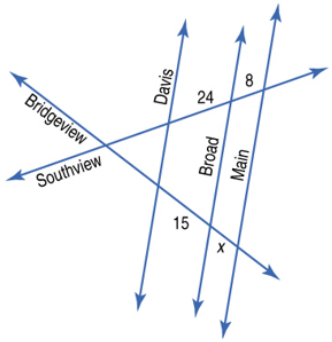
4. **REAL ESTATE** *Frontage* is the measurement of a property's boundary that runs along the side of a particular feature such as a street, lake, ocean, or river. Find the ocean frontage for Lot A to the nearest tenth of a yard.



MAPS In the figure, Larch, Maple, and Nuthatch Streets are all parallel. The figure shows the distances in between city blocks. Find *x*.

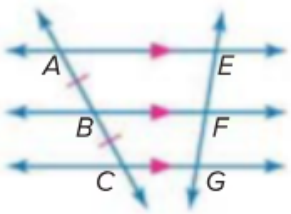


In the figure, Davis, Broad, and Main Streets are all parallel. The figure shows the distances in between city blocks. Find *x*.



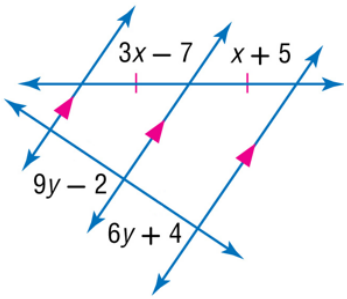
Corollary 7.2 Congruent Parts of Parallel Lines

If three or more parallel lines cut off congruent segments on one transversal, then they cut off congruent segments on every transversal.



Example If $\overline{AE} \parallel \overline{BF} \parallel \overline{CG}$, and $\overline{AB} \cong \overline{BC}$, then $\overline{EF} \cong \overline{FG}$.

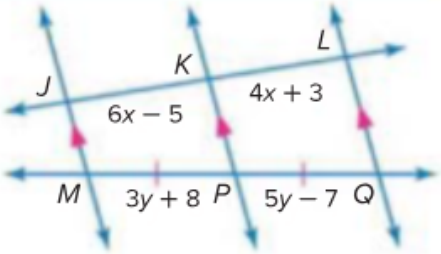
ALGEBRA **Find *x* and *y*.**



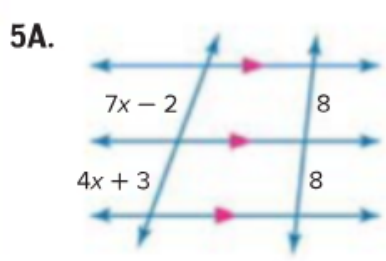
To find *x*:

ALGEBRA Find x and y.

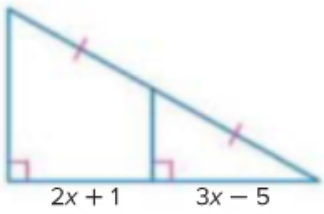
Because $\overleftrightarrow{JM} \parallel \overleftrightarrow{KP} \parallel \overleftrightarrow{LQ}$ and $\overline{MP} \cong \overline{PQ}$, then $\overline{JK} \cong \overline{KL}$ by Corollary 7.2.



5A.

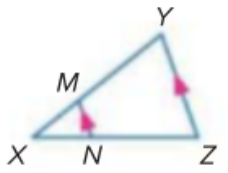


5B.



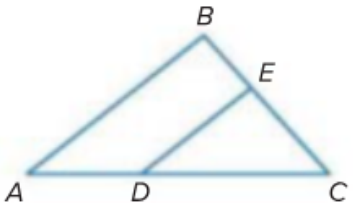
Example 1

1. If $XM = 4$, $XN = 6$, and $NZ = 9$, find XY.
2. If $XN = 6$, $XM = 2$, and $XY = 10$, find NZ.

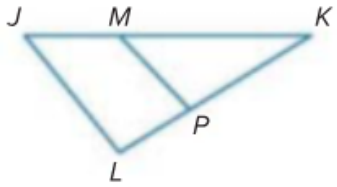


Example 2

3. In $\triangle ABC$, $BC = 15$, $BE = 6$, $DC = 12$, and $AD = 8$. Determine whether $\overline{DE} \parallel \overline{AB}$. Justify your answer.

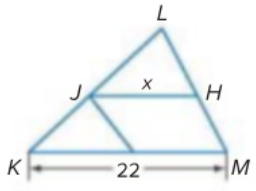


4. In $\triangle JKL$, $JK = 15$, $JM = 5$, $LK = 13$, and $PK = 9$. Determine whether $\overline{JL} \parallel \overline{MP}$. Justify your answer.



Example 3 $\overline{JH}$ **is a midsegment of** $\triangle KLM$**. Find the value of** x**.**

5.



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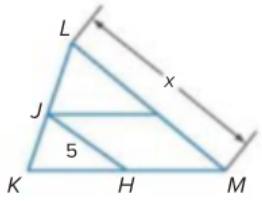
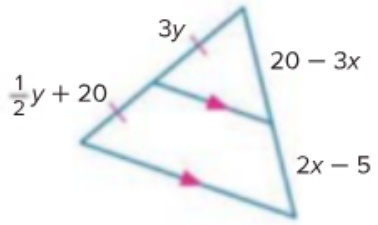


Example 4

7. MAPS Refer to the map at the right. 3rd Avenue and 5th Avenue are parallel. If the distance from 3rd Avenue to City Mall along State Street is 3201 feet, find the distance between 5th Avenue and City Mall along Union Street. Round to the nearest tenth.

Example 5 **ALGEBRA** Find x and y.

8.



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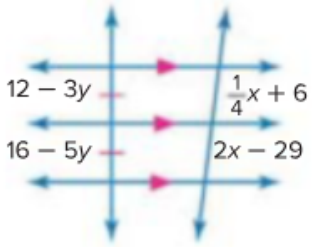
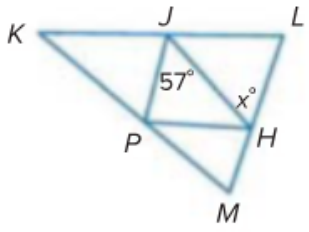


$\overline{JH}$, $\overline{JP}$, and $\overline{PH}$ are midsegments of $\triangle KLM$. Find the value of x.

18.



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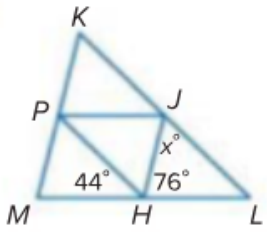

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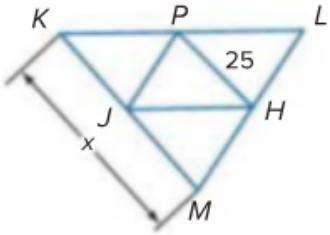

21.