6.4: The Square Root Property

$$x^2 = n \rightarrow x = \pm\sqrt{n}$$

For the square root property, we will be using both the positive and negative roots when finding a solution.

Solve:

$\sqrt{x^2} = \sqrt{16}$

$x =$

Solve:

$\sqrt{(x+6)^2} = 25$

$x + 6 = \pm 5$

$x = -6 + 5 = -1$

$x = -6 - 5 = -11$

6.4: The Square Root Property

To solve a quadratic equation using the square root property, a first step may be to factor.

Solve:

$x^2 + 10x + 25 = 49$

$(x+5)(x+5) = 49$

$\sqrt{(x+5)^2} = \pm\sqrt{49}$

$x+5 = \pm 7$

$-5 \quad -5$

$x = -5+7 = 2$

$x = -5-7 = -12$

$x^2 + 14x + 49 = 64$

$(x+7)(x+7) = 64$

$\sqrt{(x+7)^2} = \pm\sqrt{64}$

$x+7 = \pm 8$

$x = -7+8 = 1$

$x = -7-8 = -15$

Solve:

$x^2 - 6x + 9 = 32$

$(x-3)(x-3) = 32$

$(x-3)^2 = 32$

$x^2 - 8x + 16 = 20$

$(x-4)(x-4)$

$\sqrt{(x-4)^2} = \pm\sqrt{20}$

$x - 4 = \pm\sqrt{20}$

$x = 4 \pm 2\sqrt{5}$

$x^2 + 10x + 25 = 48$

$(x+5)(x+5) = 48$

$(x+5)^2 = 48$

$x^2 - 14x + 49 = 12$

$(x-7)(x-7)$

$(x-7)^2 = 12$

Notice the 'c' term

Completing the square

You can only use the square root property when the quadratic is a perfect square. Most quadratics are not perfect squares. To make a quadratic a perfect square, a method called ***completing the square*** is used.

In a quadratic that is a perfect square there is a relationship between the b term and the c term.

$$(x+7)^2$$

$$(x+7)(x+7)=$$

$$x^2+7x+7x+49=$$

$$x^2+2(7x)+49=$$

$$x^2+14x+49$$

$$\left(\frac{14}{2}\right)^2 \rightarrow 7^2$$

To complete the square for any quadratic expression of the form

$$x^2+bx+\_\_\_\_$$

Step 1 Find one half of b $\frac{b}{2}$

Step 2 Square the result in step 1 $\left(\frac{b}{2}\right)^2$

Step 3 Add the result of step 2 to $x^2+bx+\underline{\left(\frac{b}{2}\right)^2}$ $= 0+\left(\frac{b}{2}\right)^2$

6-4: Completing the Square

Find the value of c to make the trinomial a perfect square.

$x^2 + 12x + c$

$\frac{12}{2} = 6^2 = 36$

$x^2 - 18x + c$

$-\frac{18}{2} = -9^2 = 81$

$x^2 + 40x + c$ $\left(\frac{40}{2}\right)^2 = 20^2 = 400$

$x^2 + 9x + c$

$\left(\frac{9}{2}\right)^2 = \frac{81}{4}$

$x^2 - 13x + c$

$\left(-\frac{13}{2}\right)^2 = \frac{169}{4}$

$x^2 + 27x + c$

$\left(\frac{27}{2}\right)^2 = \frac{729}{4}$

6-4: Completing the Square

Now we'll combine completing the square
with the square root property to solve "non-perfect" trinomials.

$x^2 + 8x - 20 = 0$ $\frac{8}{2} = 4^2$

$x^2 + 8x + 16 = 20 + 16$

$(x+4)(x+4)$

$\sqrt{(x+4)^2} = \pm\sqrt{36}$

$x + 4 = \pm 6$

$x = -4 + 6 = 2$

$x = -4 - 6 = -10$

$x^2 + 2x - 120 = 0$ $\frac{2}{2} = 1^2$

$x^2 + 2x + 1 = 120 + 1$

$(x+1)(x+1)$

$\sqrt{(x+1)^2} = \pm\sqrt{121}$

$x + 1 = \pm 11$

$x = -1 + 11 = 10$

$x = -1 - 11 = -12$

$x = -1 \pm \sqrt{5}$

6-4: Completing the Square

Solve by completing the square:

$\frac{2}{2} = 1^2$

$x^2 + 2x - 6 = 0$

$x^2 + 2x + 1 = 6 + 1$

$\sqrt{(x+1)^2} = \pm\sqrt{7}$

$x + 1 = \pm\sqrt{7}$

$x = -1 \pm \sqrt{7}$

$\frac{16}{2} = 8^2$

$x^2 + 16x - 7 = 0$

$x^2 + 16x + 64 = 7 + 64$

$\sqrt{(x+8)^2} = \pm\sqrt{71}$

$x + 8 = \pm\sqrt{71}$

$x = -8 \pm \sqrt{71}$