# Section 12-4 Expected Value

Expected value is used to determine the expected results of an experiment or business venture over the long term.

Where it is used:

1. Business - to predict future profits on new products

2. Insurance - to determine how much each policy should cost for the company to make a profit.

3. Gaming - to predict the expected gain or loss (on average) in games like lottery craps, slot machines, and roulette.

Example:

Tim tells Barbara that he will give her $1 if she can roll an even number on a single die. If she rolls an odd number on the die then Barbara must give Tim $1. Who will win money over the long run if this game were played many times?

Tim's expected gains or loss = P(Tim wins)•(amount Tim wins) +P(Tim loses)•(amount Tim loses)

$$\frac{1}{2} \cdot (1) + \frac{1}{2}(-1)$$

$$= \frac{1}{2} + -\frac{1}{2} = 0$$

This is a fair game because the expected value is zero.

If the expected value is positive it is a gain.
If the expected value is negative it is a loss.

The expected value, E, is calculated by multiplying the probability of an event occurring by the net amount gained or lost if the event occurs. If there are a number of different events and amounts to be considered, we use the following formula.

**EXPECTED VALUE**

$$E = P_1 \cdot A_1 + P_2 \cdot A_2 + P_3 \cdot A_3 + \cdots + P_n \cdot A_n$$

## EXAMPLE 1 *A New Business Venture*

JetBlue Airways is considering adding a route to the city of Minneapolis, Minnesota. Before the company makes its decision as to whether or not to service Minneapolis, it needs to consider many factors, including potential profits and losses. Factors that may affect the company's profits and losses include the number of competing airlines, the potential number of customers, the overhead costs, and fees it must pay. After considerable research, the company estimates that if it serves Minneapolis, there is a 60% chance of making a $900,000 profit, a 10% chance of breaking even, and a 30% chance of losing $1,400,000. How much can JetBlue Airways "expect" to make on this new route?

$$E(x) = .60(900{,}000) + .10(0) + .30(-1{,}400{,}000)$$

$$= 120{,}000$$

## EXAMPLE 2 *Test-Taking Strategy*

Maria is taking a multiple-choice exam in which there are five possible answers f each question. The instructions indicate that she will be awarded 2 points for eac correct response, that she will lose $\frac{1}{2}$ point for each incorrect response, and that n points will be added or subtracted for answers left blank.

a) If Maria does not know the correct answer to a question, is it to her advantage disadvantage to guess at an answer?

b) If she can eliminate one of the possible choices, is it to her advantage or disac vantage to guess at the answer?

a) $E(x) = \frac{1}{5}(2) + \frac{4}{5}\left(-\frac{1}{2}\right) = 0$

b) $E(x) = \frac{1}{4}(2) + \frac{3}{4}\left(-\frac{1}{2}\right) = \frac{1}{8}$ or .125

## EXAMPLE 3 *Selling Hot Dogs*

An outdoor hot dog vendor sells an average of 50 hot dogs per day in dry weather and an average of 15 per day in wet weather. If the weather in this area is wet 25% of the time, determine the expected (average) number of hot dogs sold per day.

dry wet

$E(x) = .75(50) + .25(15) = 41.25$

## EXAMPLE 4 *Winning a Door Prize*

When Josh Rosenberg attends a charity event, he is given a free ticket for the $50 door prize. A total of 100 tickets will be given out. Determine his expectation of winning the door prize.

win

$E(x) = \frac{1}{100}(50) + \frac{99}{100}(0)$:

$= \frac{1}{2} = .50\,¢$

**EXAMPLE 5** *Winning a Door Prize*

When Josh Rosenberg attends a charity event, he is given the opportunity to purchase a ticket for the \$50 door prize. The cost of the ticket is \$2, and 100 tickets will be sold. Determine Josh's expectation if he purchases one ticket.

$$E(x) = \frac{1}{100}(48) + \frac{99}{100}(-2) = \$-1.50$$