

Section 6.3 Test for Parallelograms

Then

You recognized and applied properties of parallelograms.

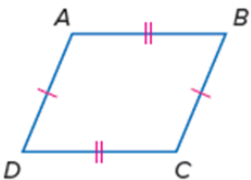
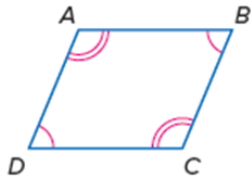
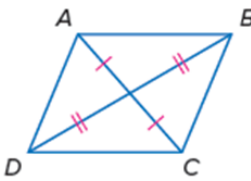
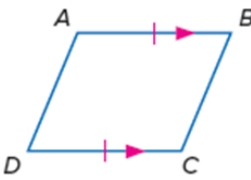
Now

- Recognize the conditions that ensure a quadrilateral is a parallelogram.
- Prove that a set of points forms a parallelogram in the coordinate plane.

Section 6.3 Test for Parallelograms

1 Conditions for Parallelograms If a quadrilateral has each pair of opposite sides parallel, it is a parallelogram by definition.

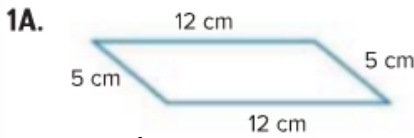
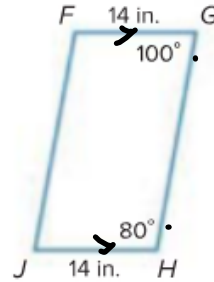
This is not the only test, however, that can be used to determine if a quadrilateral is a parallelogram.

Theorems Conditions for Parallelograms	
<p>6.9 If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.</p> <p>Abbreviation <i>If both pairs of opp. sides are \cong, then quad. is a \square.</i></p> <p>Example If $\overline{AB} \cong \overline{DC}$ and $\overline{AD} \cong \overline{BC}$, then $ABCD$ is a parallelogram.</p>	
<p>6.10 If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.</p> <p>Abbreviation <i>If both pairs of opp. \angles are \cong, then quad. is a \square.</i></p> <p>Example If $\angle A \cong \angle C$ and $\angle B \cong \angle D$, then $ABCD$ is a parallelogram.</p>	
<p>6.11 If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.</p> <p>Abbreviation <i>If diag. bisect each other, then quad. is a \square.</i></p> <p>Example If \overline{AC} and \overline{DB} bisect each other, then $ABCD$ is a parallelogram.</p>	
<p>6.12 If one pair of opposite sides of a quadrilateral is both parallel and congruent, then the quadrilateral is a parallelogram.</p> <p>Abbreviation <i>If one pair of opp. sides is \cong and \parallel, then the quad. is a \square.</i></p> <p>Example If $\overline{AB} \parallel \overline{DC}$ and $\overline{AB} \cong \overline{DC}$, then $ABCD$ is a parallelogram.</p>	

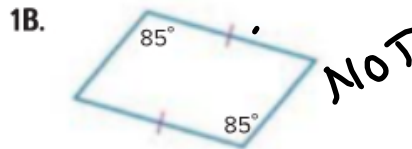
Example 1 Identify Parallelograms

Determine whether the quadrilateral is a parallelogram. Justify your answer.

yes opp sides are both \parallel and \cong

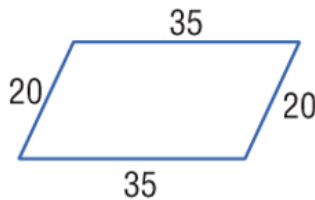


yes both opp sides \cong



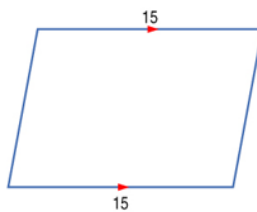
NOT

Determine whether the quadrilateral is a parallelogram. Justify your answer.



yes both opp sides are \cong

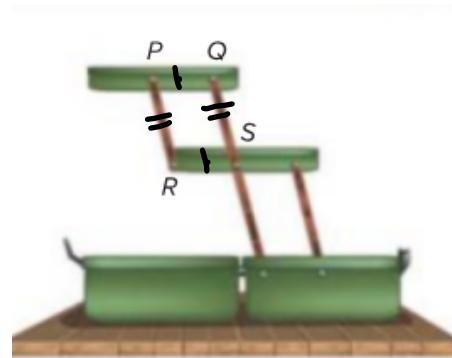
Which method would prove the quadrilateral is a parallelogram?



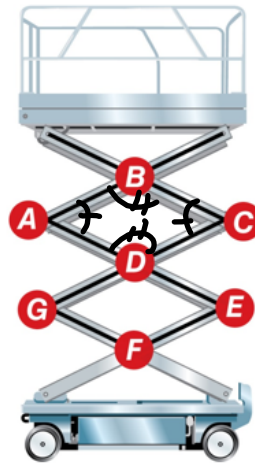
yes opp sides are both \parallel and \cong

FISHING The diagram shows a side view of the tackle box at the left. In the diagram, $PQ = RS$ and $PR = QS$. Explain why the upper and middle trays remain parallel no matter to what height the trays are raised or lowered.

both pairs of opp sides are \cong

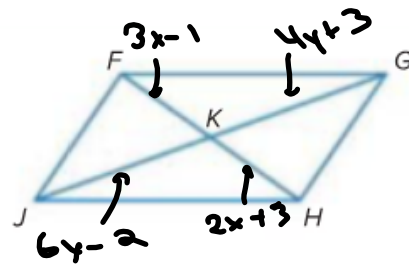


MECHANICS Scissor lifts, like the platform lift shown, are commonly applied to tools intended to lift heavy items. In the diagram, $\angle A \cong \angle C$ and $\angle B \cong \angle D$. Explain why the consecutive angles will always be supplementary regardless of the height of the platform.



both opp. \angle s are \cong
 \therefore it's a \square
 Consecutive \angle s are supplementary

If $FK = 3x - 1$, $KG = 4y + 3$, $JK = 6y - 2$, and $KH = 2x + 3$, find x and y so that the quadrilateral is a parallelogram.



$$3x - 1 = 2x + 3$$

$$x = 4$$

$$6y - 2 = 4y + 3$$

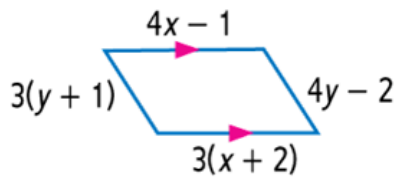
$$2y = 5$$

$$y = \frac{5}{2}$$

$$3 \cdot 6 \left(\frac{5}{2}\right) - 2 = 7 \left(\frac{5}{2}\right) + 3$$

$$13 = 13$$

Find x and y so that the quadrilateral is a parallelogram.



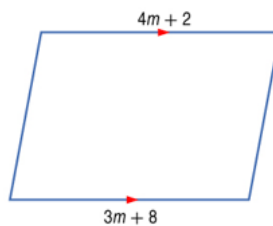
$$4x - 1 = 3x + 6$$

$$x = 7$$

$$3y + 3 = 4y - 2$$

$$5 = y$$

Find m so that the quadrilateral is a parallelogram.

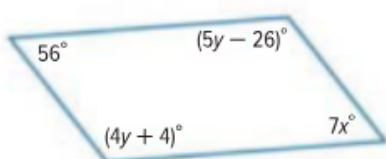


$$4m + 2 = 3m + 8$$

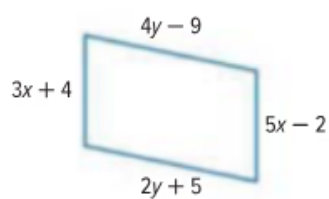
$$m = 6$$

Find x and y so that each quadrilateral is a parallelogram.

3A.



3B.



COORDINATE GEOMETRY Graph quadrilateral $KLMN$ with vertices $K(2, 3)$, $L(8, 4)$, $M(7, -2)$, and $N(1, -3)$. Determine whether the quadrilateral is a parallelogram. Justify your answer using the Slope Formula.

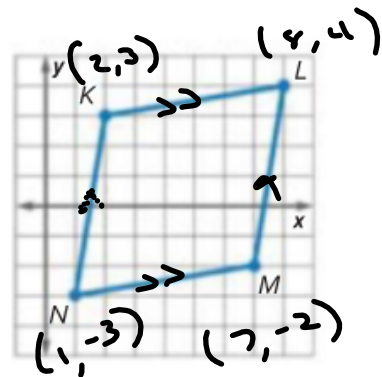
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\begin{array}{c} KN \\ \frac{3 - (-3)}{2 - 1} = \frac{6}{1} = 6 \end{array}$$

$$\begin{array}{c} LM \\ \frac{4 - (-2)}{8 - 7} = \frac{6}{1} = 6 \end{array}$$

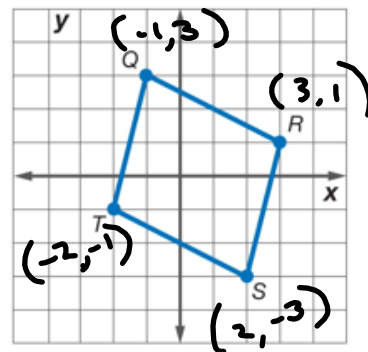
$$\begin{array}{c} KL \\ \frac{4 - 3}{8 - 2} = \frac{1}{6} \end{array}$$

$$\begin{array}{c} NM \\ \frac{-2 - (-3)}{7 - 1} = \frac{1}{6} \end{array}$$



COORDINATE GEOMETRY

Graph Quadrilateral $QRST$ has vertices $Q(-1, 3)$, $R(3, 1)$, $S(2, -3)$, and $T(-2, -1)$. Determine whether the quadrilateral is a parallelogram. Justify your answer by using the Slope Formula.



$$\begin{array}{c} QT \\ \frac{3 - (-1)}{-1 - (-2)} = \frac{4}{1} \end{array}$$

$$\begin{array}{c} RS \\ \frac{1 - (-3)}{3 - 2} = \frac{4}{1} \end{array}$$

$$\begin{array}{c} QR \\ \frac{1 - 3}{3 - (-1)} = \frac{-2}{4} \end{array}$$

$$\begin{array}{c} TS \\ \frac{-3 - (-1)}{2 - (-2)} = \frac{-2}{4} \end{array}$$

Graph quadrilateral $EFGH$ with vertices $E(-2, 2)$, $F(2, 0)$, $G(1, -5)$, and $H(-3, -2)$. Determine whether the quadrilateral is a parallelogram.

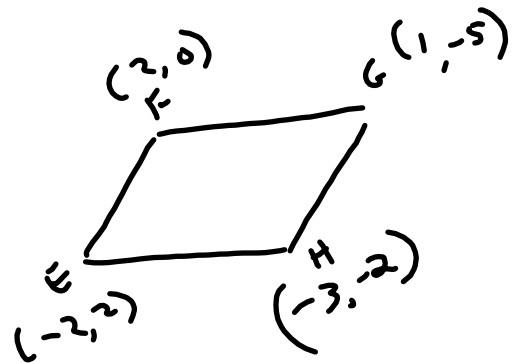
$$EF \quad \frac{0-2}{2-(-2)} = -\frac{2}{4}$$

$$FG \quad \frac{-5-0}{1-2} = \frac{-5}{-1} = \frac{5}{1}$$

$$GH \quad \frac{-5-(-2)}{1-(-3)} = \frac{-3}{4}$$

$$HE \quad \frac{-2-2}{-3-(-2)} = \frac{-4}{-1}$$

NOT



Concept Summary

Prove that a Quadrilateral Is a Parallelogram

- Show that both pairs of opposite sides are parallel. (Definition)
- Show that both pairs of opposite sides are congruent. (Theorem 6.9)
- Show that both pairs of opposite angles are congruent. (Theorem 6.10)
- Show that the diagonals bisect each other. (Theorem 6.11)
- Show that a pair of opposite sides is both parallel and congruent. (Theorem 6.12)

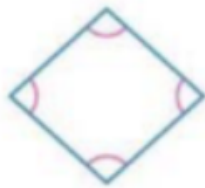
Check Your Understanding



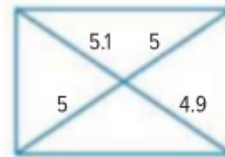
= Step-by-Step Solutions begin on page R13.

Example 1 Determine whether each quadrilateral is a parallelogram. Justify your answer.

1.

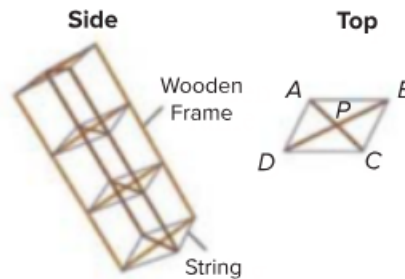


2.



Example 2

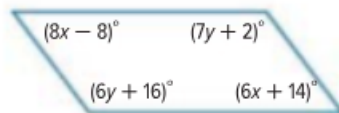
3. **KITES** Charmaine is building the kite shown below. She wants to be sure that the string around her frame forms a parallelogram before she secures the material to it. How can she use the measures of the wooden portion of the frame to prove that the string forms a parallelogram? Explain your reasoning.



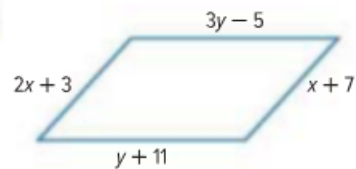
Example 3

ALGEBRA Find x and y so that the quadrilateral is a parallelogram.

4.



5

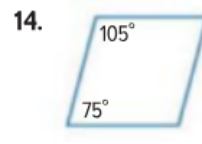
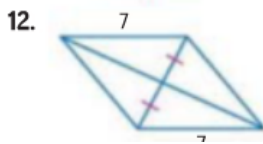
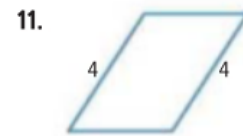
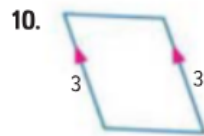
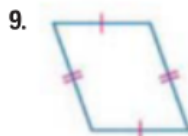


Example 4 **COORDINATE GEOMETRY** Graph each quadrilateral with the given vertices. Determine whether the figure is a parallelogram. Justify your answer with the method indicated.

6. $A(-2, 4)$, $B(5, 4)$, $C(8, -1)$, $D(-1, -1)$; Slope Formula

Practice and Problem Solving Extra Practice is on page R6.

Example 1 **MP CONSTRUCT ARGUMENTS** Determine whether each quadrilateral is a parallelogram. Justify your answer.



ALGEBRA Find x and y so that the quadrilateral is a parallelogram.

