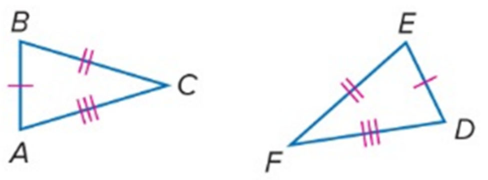
Section 4.3

Proving Triangles Congruent–SSS, SAS

Postulate 4.1 Side-Side-Side (SSS) Congruence

If three sides of one triangle are congruent to three sides of a second triangle, then the triangles are congruent.

Example If **S**ide $\overline{AB} \cong \overline{DE}$,
Side $\overline{BC} \cong \overline{EF}$, and
Side $\overline{AC} \cong \overline{DF}$,
then $\triangle ABC \cong \triangle DEF$.

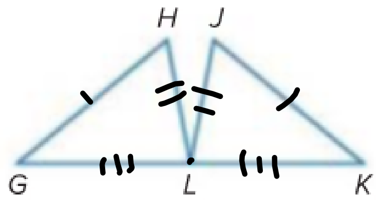


Example 1 Use SSS to Prove Triangles Congruent

Write a flow proof.

Given: $\overline{GH} \cong \overline{KJ}$, $\overline{HL} \cong \overline{JL}$, and L is the midpoint of $\overline{GK}$.

Prove: $\triangle GHL \cong \triangle KJL$



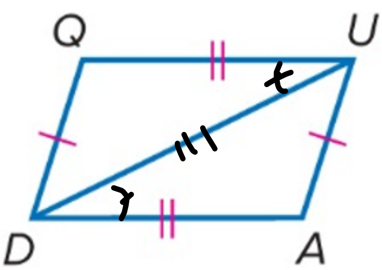
1. | 1. Given

2. $\overline{GL} \cong \overline{LK}$ | 2. Def of midpt.

3. $\triangle GHL \cong \triangle KJL$ | 3. SSS Post.

Given: $\overline{QU} \cong \overline{AD}$, $\overline{QD} \cong \overline{AU}$

Prove: ~~$\triangle QUD \cong \triangle ADU$~~ $\angle Q \cong \angle A$



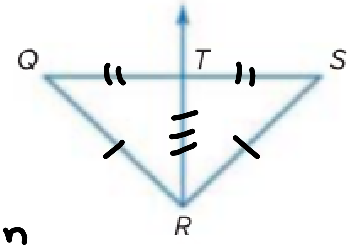
1. | 1. Given

2. $\overline{DU} \cong \overline{UD}$ | 2. Reflective Prop

3. $\triangle QUD \cong \triangle ADU$ | 3. SSS Post.
   $\angle Q \cong \angle A$ | 4. CPCTC

Given: $\triangle QRS$ is isosceles with $\overline{QR} \cong \overline{SR}$.
$\overline{RT}$ bisects $\overline{QS}$ at point T.

Prove: $\triangle QRT \cong \triangle SRT$



1. 1. Given
2. QT ≅ TS 2. Def of seg. bisector
3. RT ≅ RT 3. Reflective
4. △QRT ≅ △SRT 4. SSS Post

Example 2 SSS on the Coordinate Plane

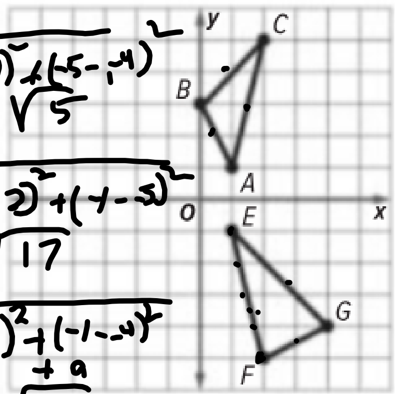
Triangle ABC has vertices $A(1, 1)$, $B(0, 3)$, and $C(2, 5)$. Triangle EFG has vertices $E(1, -1)$, $F(2, -5)$, and $G(4, -4)$.

a. Graph both triangles on the same coordinate plane.

b. Use your graph to make a conjecture as to whether the triangles are congruent. Explain your reasoning using rigid motions.

c. Write a logical argument using coordinate geometry to support the conjecture you made in part b.

a.



$\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

AB $\sqrt{(1-0)^2+(1-3)^2}=\sqrt{5}$.

AC $\sqrt{(1-2)^2+(1-5)^2}$ $\sqrt{17}$

BC $\sqrt{(0-2)^2+(3-5)^2}$ $\sqrt{8}$

FG $\sqrt{(2-4)^2+(-5-(-4))^2}$ $\sqrt{5}$

FE $\sqrt{(1-2)^2+(-1-(-5))^2}$ $\sqrt{17}$

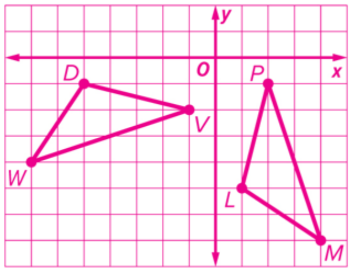
EG $\sqrt{(1-4)^2+(-1-(-4))^2}$ $9+9$ $\sqrt{18}$

Triangle *DVW* has vertices *D* (–5, –1), *V* (–1, –2), and *W* (–7, –4). Triangle *LPM* has vertices *L* (1, –5), *P* (2, –1), and *M* (4, –7).

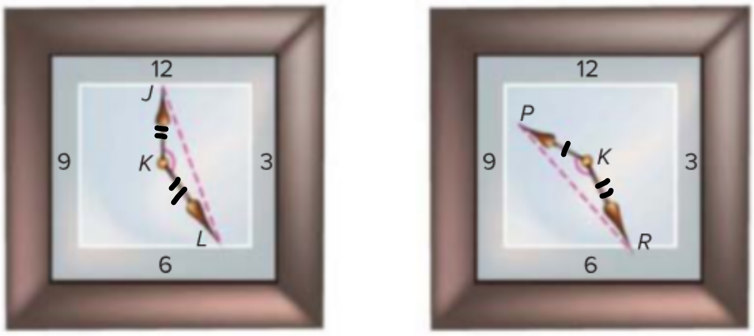
A. Graph both triangles on the same coordinate plane.

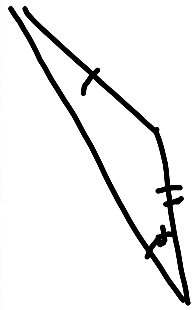
B. Use your graph to make a conjecture as to whether the triangles are congruent. Explain your reasoning using rigid motions.

C. Write a logical argument that uses coordinate geometry to support the conjecture you made in part B.



2 SAS Postulate The angle formed by two adjacent sides of a polygon is called an **included angle**. Consider included angle *JKL* formed by the hands on the first clock shown below. Any time the hands form an angle with the same measure, the distance between the ends of the hands $\overline{JL}$ and $\overline{PR}$ will be the same.





$\triangle PKR \cong \triangle JKL$

Postulate 4.2 Side-Angle-Side (SAS) Congruence

Words If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then the triangles are congruent.

Example If **S**ide $\overline{AB} \cong \overline{DE}$,
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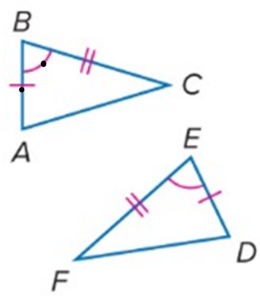

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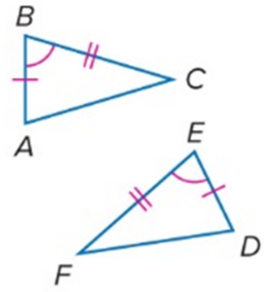
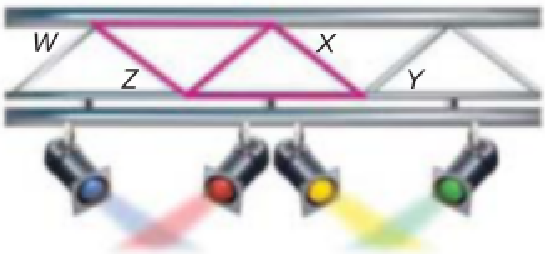


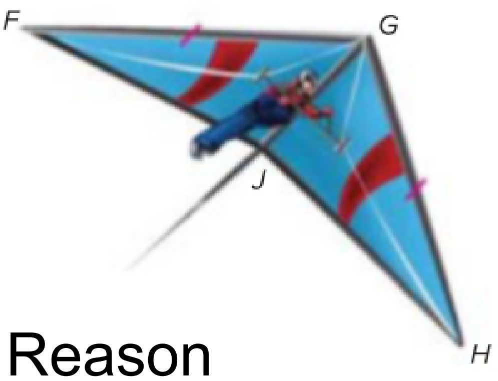
Real-World Example 3 Use SAS to Prove Triangles are Congruent

LIGHTING The scaffolding for stage lighting shown appears to be made up of congruent triangles. If $\overline{WX} \cong \overline{YZ}$ and $\overline{WX} \parallel \overline{ZY}$, write a two-column proof to prove that $\triangle WXZ \cong \triangle YZX$.



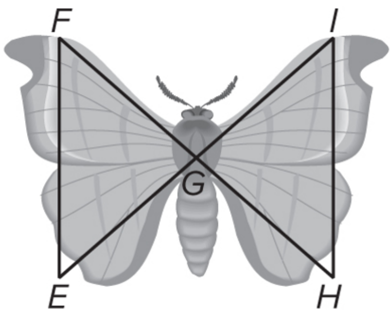
Statement Reason

EXTREME SPORTS The wings of the hang glider shown appear to be congruent triangles. If $\overline{FG} \cong \overline{GH}$ and $\overline{JG}$ bisects $\angle FGH$, prove that $\triangle FGJ \cong \triangle HGJ$.



Statement Reason

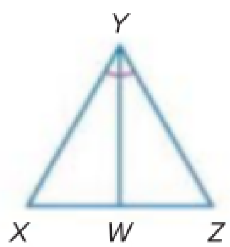
Entomology **The wings of one type of moth form two triangles. Write a two-column proof to prove that $\triangle FEG \cong \triangle HIG$ if $\overline{EI} \cong \overline{FH}$, and G is the midpoint of both $\overline{EI}$ and $\overline{FH}$.**



Statement	Reason

Given: $\triangle XYZ$ is equilateral.
$\overline{WY}$ bisects $\angle XYZ$.

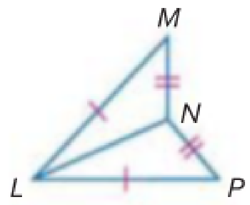
Prove: $\overline{XW} \cong \overline{ZW}$



Write a two-column proof.

Given: $\overline{MN} \cong \overline{PN}, \overline{LM} \cong \overline{LP}$

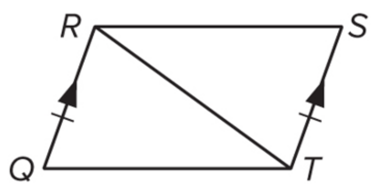
Prove: $\angle LNM \cong \angle LNP$



Statement Reason

Given: $\overline{RQ} \parallel \overline{TS}$
$\overline{RQ} \cong \overline{TS}$

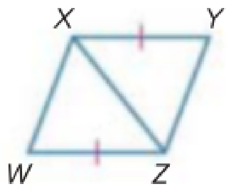
Prove: $\angle Q \cong \angle S$



Statement Reason

Given: $\overline{YX} \cong \overline{WZ}, \overline{YX} \parallel \overline{ZW}$

Prove: $\triangle YXZ \cong \triangle WZX$



CONSTRUCT ARGUMENTS Determine which postulate can be used to prove that the triangles are congruent. If it is not possible to prove congruence, write *not possible*. If it is possible, describe the rigid motions that map one triangle onto the other.

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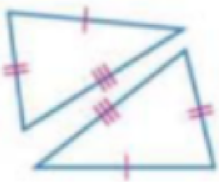

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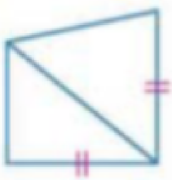

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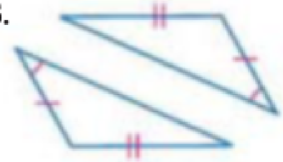

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