

Section 2.2 Subsets

**Subset** - when we break larger sets into smaller more manageable sets

Set A is a **subset** of set B, symbolized by  $A \subseteq B$ , if and only if all the elements of set A are also elements of set B.

Example Determine whether set A is a subset of set B

a)  $A = \{\text{marigold, pansy, geranium}\}$   
 $B = \{\text{marigold, pansy, begonia, geranium}\}$       $A \subseteq B$       $B \not\subseteq A$

b)  $A = \{2,3,4,5\}$   
 $B = \{2,3\}$      NO      $A \not\subseteq B$       $B \subseteq A$

c)  $A = \{x/x \text{ is a yellow fruit}\}$   
 $B = \{X/X \text{ is a red fruit}\}$       $A \not\subseteq B$       $B \not\subseteq A$

d)  $A = \{\}$   
 $B = \{2,3,4\}$       $A \subseteq B$

The  $\emptyset$  is a subset of ANY SET

**Proper subset** - Set A is a proper subset of set B, symbolized  $A \subset B$ , if and only if all the elements of set A are elements of set B and  $A \neq B$ . (set B must contain at least one element not in set A)

Example: Determine if set A is a proper subset of set B

a)  $A = \{\text{jazz, pop, hip hop}\}$   $A \subset B$   
 $B = \{\text{classical, jazz, pop, rap, hip hop}\}$

b)  $A = \{a, b, c, d\}$   ~~$A \subset B$~~   $A \subseteq B$   
 $B = \{a, c, b, d\}$

### Example: Elements or Subset?

Determine whether the following are true or false.

- a)  $3 \in \{3, 4, 5\}$  True
- b)  $\{3\} \in \{3, 4, 5\}$  False b/c  $\{ \neq \}$
- c)  $\{3\} \in \{\{3\}, \{4\}, \{5\}\}$  True
- d)  $\{3\} \subseteq \{3, 4, 5\}$  True
- e)  $3 \subseteq \{3, 4, 5\}$  False need  $\{ \}$   $\{3\}$
- f)  $\{ \} \subseteq \{3, 4, 5\}$  True

| Set       | Subsets   | Number of Subsets             |
|-----------|---|-------------------------------|
| {}        | $\{\}$  | 1 $2^0$                       |
| {a}       | $\{\}$ $\{a\}$  | 2 $2^1$                       |
| {a, b}    | $\{\}$ $\{a\}$ $\{b\}$ $\{a, b\}$   | 4 $2^2$                       |
| {a, b, c} | $\{\}$ $\{a\}$ $\{b\}$ $\{c\}$ $\{a, b\}$ $\{a, c\}$ $\{b, c\}$ $\{a, b, c\}$ | 8 $2^3$<br>16<br>32<br>4<br>5 |

$2^{10}$

The **number of distinct subsets** of a finite set A is  $2^n$ , where  $n$  is the number of elements in set A.

Example Distinct Subsets

- a) Determine the number of distinct subsets for the set {B,O, A, T}  $2^4 = 16$
- b) List all the distinct subsets for the set {B, O, A, T}
- c) How many of the distinct subsets are proper subsets. 15

$\{ \}$   $\{B\}$   $\{O\}$   $\{A\}$   $\{T\}$   
 $\{BO\}$   $\{BA\}$   $\{BT\}$   $\{OA\}$   $\{OT\}$   $\{AT\}$   
 $\{BOA\}$   $\{BOT\}$   $\{OAT\}$   $\{BAT\}$   
 $\{BOAT\}$

Proper Sub.  $2^n - 1$

Example

Michael Bolton is going to purchase a pizza that he can share with the entire band. To his cheese pizza, he can add any of the following toppings: pepperoni, sausage, onions, mushrooms, anchovies, and ham. How many different variations of the pizza and toppings can be made?

$$2^6 = 64$$