

Section 4.2 Congruent Triangles

Congruence and Corresponding Parts

In two congruent Polygons, all of the parts of one polygon are congruent to the corresponding parts or matching parts of the other polygon. These corresponding parts include corresponding angles and corresponding sides.

Key Concept		Definition of Congruent Polygons
Words	Two polygons are congruent if and only if their corresponding parts are congruent. Two polygons are congruent if and only if a rigid motion or a series of rigid motions maps one polygon exactly onto the other.	Model
Example	<p>Corresponding Angles</p> $\angle A \cong \angle H$ $\angle B \cong \angle J$ $\angle C \cong \angle K$	
	<p>Corresponding Sides</p> $\overline{AB} \cong \overline{HJ}$ $\overline{BC} \cong \overline{JK}$ $\overline{AC} \cong \overline{HK}$	
	<p>Congruence Statement</p> $\triangle ABC \cong \triangle HJK$	

HJK

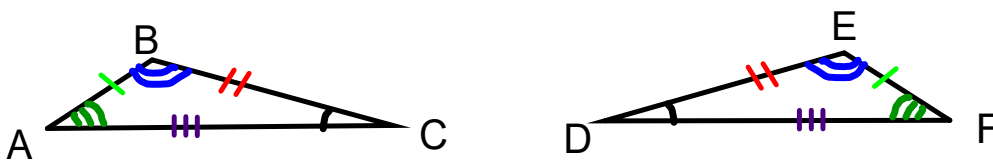
Valid congruence statements list the corresponding vertices in the same order.

$$\triangle CBA \cong \triangle KJH$$

$$\overline{BA} \cong \overline{JH}$$

$$\angle A \cong \angle H$$

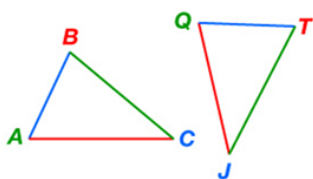
Congruent polygons - have corresponding sides congruent and corresponding angles congruent.



List the corresponding sides and vertices, then write a congruence statement.

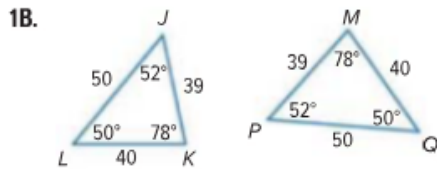
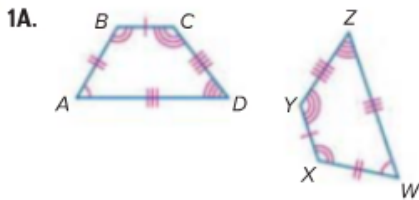
$$\begin{array}{l} \overline{AB} \rightarrow \overline{EF} \\ \overline{BC} \rightarrow \overline{DE} \\ \overline{AC} \rightarrow \overline{FD} \end{array} \quad \begin{array}{l} \angle A \rightarrow \angle F \\ \angle B \rightarrow \angle E \\ \angle C \rightarrow \angle D \end{array} \quad \triangle ABC \cong \triangle FED$$

**1 EXAMPLE**  $\triangle ABC \cong \triangle QTJ$ . List the congruent corresponding parts.



$$\begin{array}{l} AB \rightarrow QT \\ BC \rightarrow TJ \\ CA \rightarrow JQ \end{array} \quad \begin{array}{l} \angle A \rightarrow \angle Q \\ \angle B \rightarrow \angle T \\ \angle C \rightarrow \angle J \end{array}$$

List the corresponding sides and vertices, then write a congruence statement.



$\angle J \rightarrow \angle P$        $JK \rightarrow MP$   
 $\angle K \rightarrow \angle Q$        $KL \rightarrow MQ$   
 $\angle L \rightarrow \angle M$        $LJ \rightarrow QP$   
 $\triangle JKL \cong \triangle PMQ$

If two polygons are congruent, then all their corresponding parts are congruent.

For triangle, we say *Corresponding Parts of Congruent Triangles are Congruent*, also known as CPCTC.

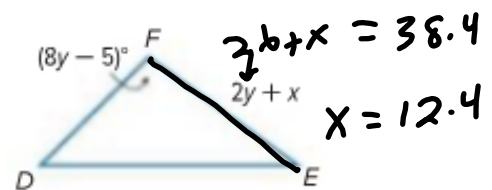
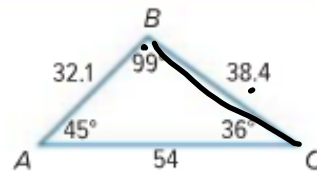
Example

In the diagram,  $\triangle ABC \cong \triangle DFE$ . Find the values of  $x$  and  $y$ .

$$8y - 5 = 99$$

$$\frac{8y}{8} = \frac{104}{8}$$

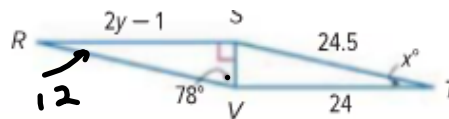
$$y = 13$$



$$2y + x = 38.4$$

$$x = 12.4$$

In the diagram,  $\triangle RSV \cong \triangle TVS$ . Find the values of  $x$  and  $y$ .



$$2y - 1 = 24$$

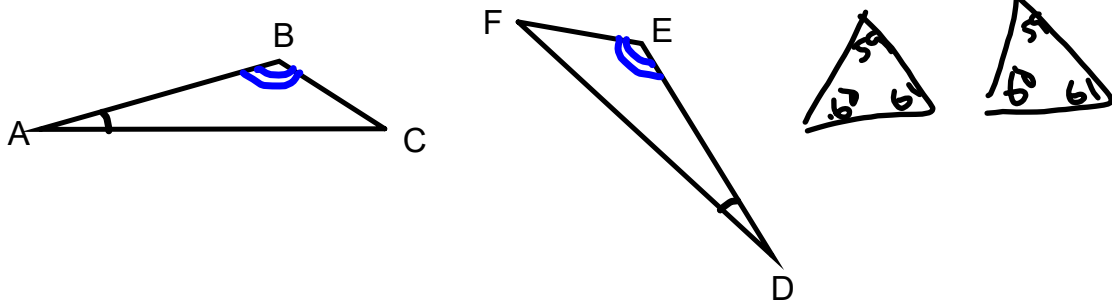
$$2y = 25$$

$$y = 12.5$$

$$x = 12$$

## Theorem 4-3 Third Angle Theorem

If 2 angles of one triangle are congruent to two angles of another triangle then the 3rd angles are congruent.

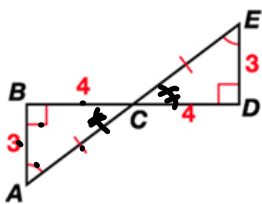
**2** EXAMPLE

$\triangle XYZ \cong \triangle KLM$ ,  $m\angle Y = 67$ , and  $m\angle M = 48$ . Find  $m\angle X$ .

$$\begin{array}{r}
 \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\
 67 \quad \quad 48 \quad \quad 67 \quad \quad 48 \\
 \\
 180 \\
 - 115 \\
 \hline
 65
 \end{array}$$

**3 EXAMPLE**

Can you conclude that  $\triangle ABC \cong \triangle CDE$  in the figure below?



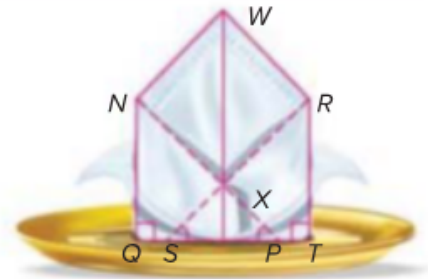
$\angle BCA \cong \angle DCE$

Third  $\angle$  Thm.

List corresponding vertices in the same order.

**Real-World Example 3** Use the Third Angles Theorem

**PARTY PLANNING** The planners of the Senior Banquet decide to fold the dinner napkins using the Triangle Pocket Fold so that they can place a small gift in the pocket. If  $\angle NPQ \cong \angle RST$ , and  $m\angle NPQ = 40$ , find  $m\angle SRT$ .



In the diagram above, if  $\angle WNX \cong \angle WRX$ ,  $\overline{WX}$  bisects  $\angle NXR$ ,  $m\angle WNX = 88$ , and  $m\angle NXW = 49$ , find  $m\angle NWR$ . Explain your reasoning.

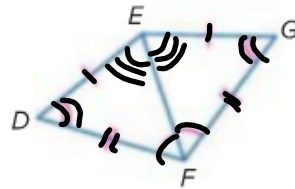
**Example 4** Prove that Two Triangles Are Congruent

Write a two-column proof.

Given:  $\overline{DE} \cong \overline{GE}$ ,  $\overline{DF} \cong \overline{GF}$ ,  $\angle D \cong \angle G$ ,  
 $\angle DFE \cong \angle GFE$

Prove:  $\triangle DEF \cong \triangle GEF$

Proof:



Statements

Reasons

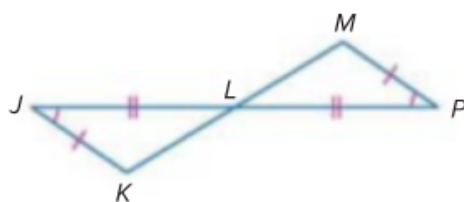
- |  |                         |
|--|-------------------------|
| 1.                                     | 1. Given                |
| 2. $\angle DEF \cong \angle GEF$       | 2. Third $\angle$ Thm   |
| 3. $\overline{FE} \cong \overline{FE}$ | 3. Reflective Prop      |
| 4. $\triangle DEF \cong \triangle GEF$ | 4. Def $\cong$ Polygons |

**Guided Practice**

4. Write a two-column proof.

**Given:**  $\angle J \cong \angle P$ ,  $\overline{JK} \cong \overline{PM}$ ,  
 $\overline{JL} \cong \overline{PL}$ , and  $L$  bisects  $\overline{KM}$ .

**Prove:**  $\triangle JLK \cong \triangle PLM$



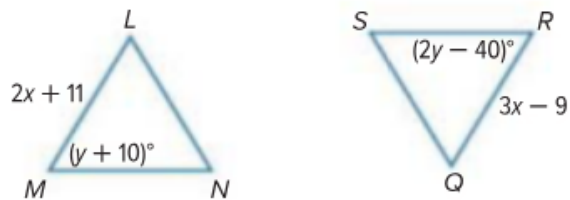


Practice

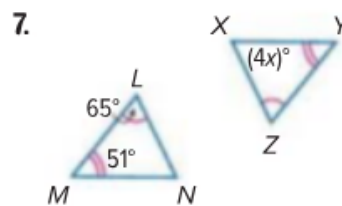
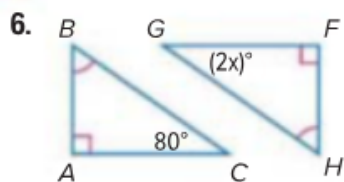
In the figure,  $\triangle LMN \cong \triangle QRS$ .

4. Find  $x$ .

5. Find  $y$ .



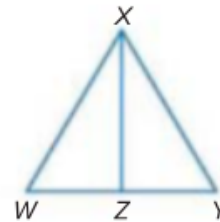
**MP REGULARITY** Find  $x$ . Explain your reasoning.



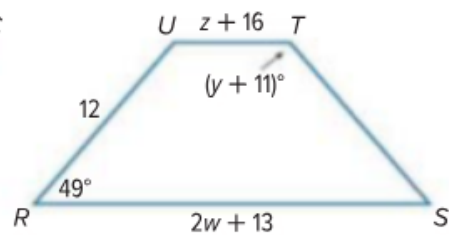
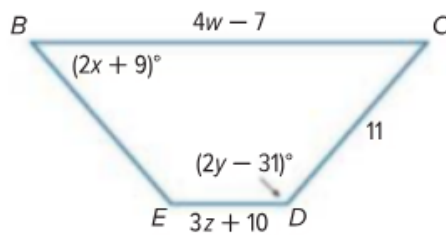
8. **PROOF** Write a paragraph proof.

**Given:**  $\angle WXZ \cong \angle YXZ$ ,  $\angle XZW \cong \angle XZY$ ,  
 $\overline{WX} \cong \overline{YX}$ ,  $\overline{WZ} \cong \overline{YZ}$

**Prove:**  $\triangle WXZ \cong \triangle YXZ$



Polygon  $BCDE \cong$  polygon  $RSTU$ . Find each value.



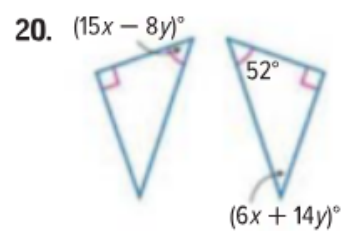
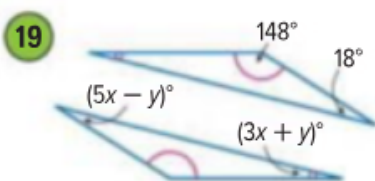
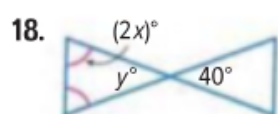
13.  $x$

14.  $y$

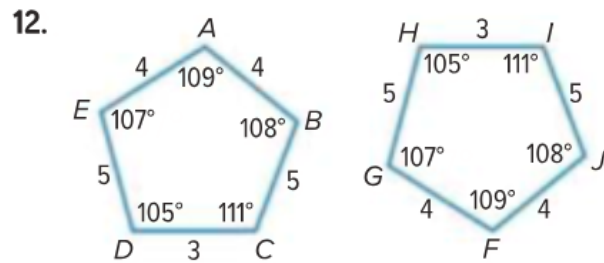
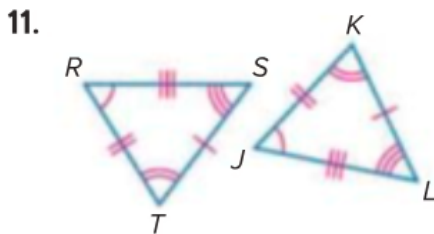
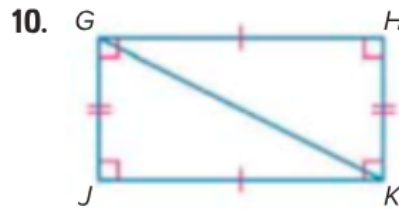
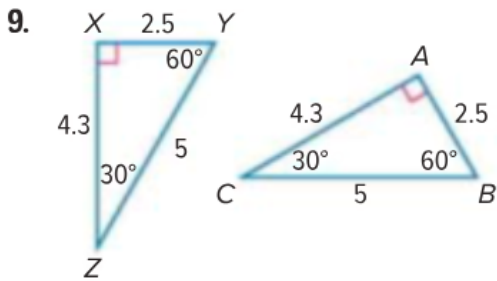
15.  $z$

16.  $w$

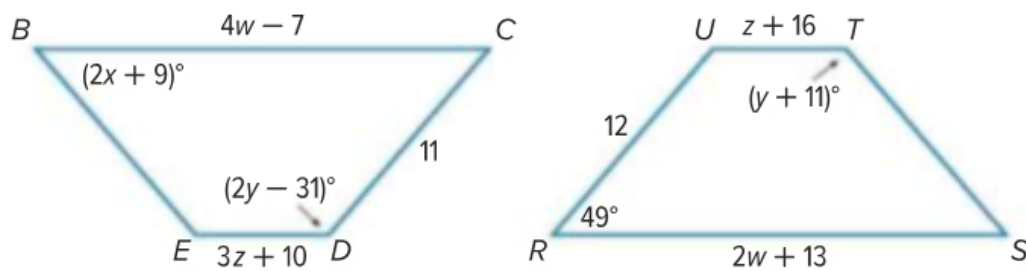
Find  $x$  and  $y$ .



Show that the polygons are congruent by using rigid motions and by identifying all congruent corresponding parts. Then write a congruence statement.



Polygon  $BCDE \cong$  polygon  $RSTU$ . Find each value.

13.  $x$ 14.  $y$ 15.  $z$ 16.  $w$ 

**ALGEBRA** Draw and label a figure to represent the congruent triangles.

Then find  $x$  and  $y$ .

28.  $\triangle ABC \cong \triangle DEF$ ,  $AB = 7$ ,  $BC = 9$ ,  $AC = 11 + x$ ,  $DF = 3x - 13$ , and  $DE = 2y - 5$

29.  $\triangle LMN \cong \triangle RST$ ,  $m\angle L = 49$ ,  $m\angle M = 10y$ ,  $m\angle S = 70$ , and  $m\angle T = 4x + 9$

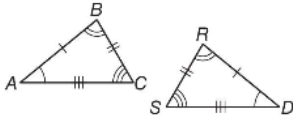
30.  $\triangle JKL \cong \triangle MNP$ ,  $JK = 12$ ,  $LJ = 5$ ,  $PM = 2x - 3$ ,  $m\angle L = 67$ ,  $m\angle K = y + 4$  and  $m\angle N = 2y - 15$

NAME \_\_\_\_\_ DATE \_\_\_\_\_ PERIOD \_\_\_\_\_

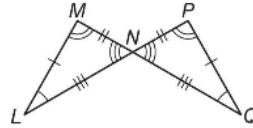
### 4-2 Practice Congruent Triangles

Show that the polygons are congruent by identifying all congruent corresponding parts. Then write a congruence statement.

1.

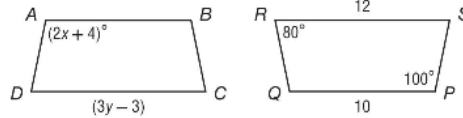


2.



Polygon  $ABCD \cong$  polygon  $PQRS$ .

3. Find the value of  $x$ .



4. Find the value of  $y$ .

5. **PROOF** Write a two-column proof.

**Given:**  $\square P \cong \square R$ ,  $\square PSQ \cong \square RSQ$ ,  $\overline{PQ} \cong \overline{RQ}$ ,

$\overline{PS} \cong \overline{RS}$

**Prove:**  $\triangle PQS \cong \triangle RQS$

