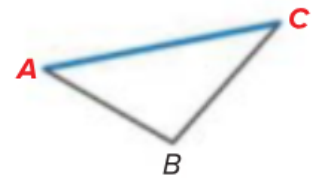


## Proving Triangles Congruent—ASA, AAS

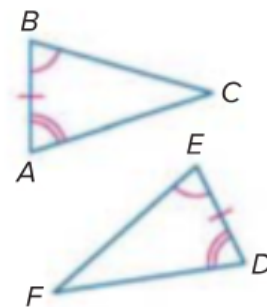
**1 ASA Postulate** An **included side** is the side located between two consecutive angles of a polygon. In  $\triangle ABC$  at the right,  $\overline{AC}$  is the included side between  $\angle A$  and  $\angle C$ .



### Postulate 4.3 Angle-Side-Angle (ASA) Congruence

If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.

**Example** If **A**ngle  $\angle A \cong \angle D$ ,  
**S**ide  $\overline{AB} \cong \overline{DE}$ , and  
**A**ngle  $\angle B \cong \angle E$ ,  
 then  $\triangle ABC \cong \triangle DEF$ .



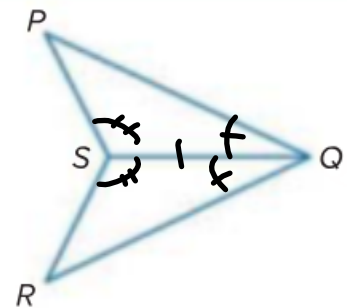
**Example 1** Use ASA to Prove Triangles Congruent

Write a two-column proof.

**Given:**  $\overline{QS}$  bisects  $\angle PQR$ .  
 $\angle PSQ \cong \angle RSQ$

**Prove:**  $\triangle PQS \cong \triangle RQS$

**Proof:**



**Statements**

**Reasons**

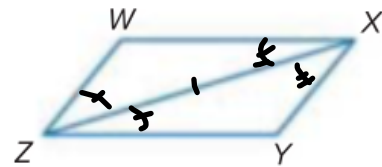
- 1.
2.  $\angle PQS \cong \angle RQS$
3.  $\overline{SQ} \cong \overline{SQ}$
4.  $\triangle PQS \cong \triangle RQS$

1. Given
2. Def of  $\angle$  bisector
3. Reflexive Prop
4. ASA Post

1. Write a flow proof.

Given:  $\overline{ZX}$  bisects  $\angle WZY$ ;  $\overline{XZ}$  bisects  $\angle YXW$ .

Prove:  $\triangle WXZ \cong \triangle YXZ$



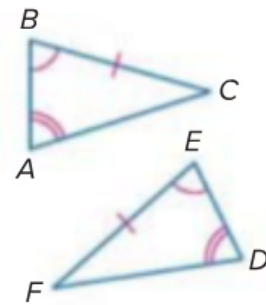
Statements	Reasons
1.	1. Given
2. $\angle WZX \cong \angle YZX$ $\angle WXZ \cong \angle YXZ$	2. Def of $\angle$ bisector
3. $\overline{ZX} \cong \overline{ZX}$	3. Reflective Prop.
4. $\triangle WXZ \cong \triangle YXZ$	4. ASA Post

**2 AAS Theorem** The congruence of two angles and a nonincluded side are also sufficient to prove two triangles congruent. This congruence relationship is a theorem because it can be proved using the Third Angles Theorem.

### Theorem 4.5 Angle-Angle-Side (AAS) Congruence

If two angles and the nonincluded side of one triangle are congruent to the corresponding two angles and side of a second triangle, then the two triangles are congruent.

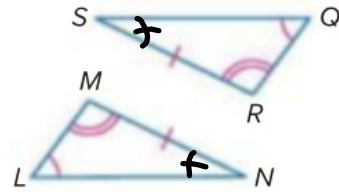
**Example** If **Angle**  $\angle A \cong \angle D$ ,  
**Angle**  $\angle B \cong \angle E$ , and  
**Side**  $\overline{BC} \cong \overline{EF}$ ,  
then  $\triangle ABC \cong \triangle DEF$ .



**Proof Angle-Angle-Side Theorem**

Given:  $\angle L \cong \angle Q, \angle M \cong \angle R, \overline{MN} \cong \overline{RS}$

Prove:  $\triangle LMN \cong \triangle QRS$

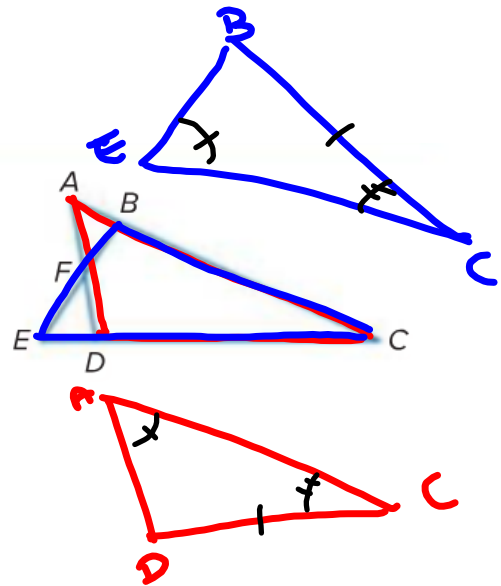


- 1.
  2.  $\angle N \cong \angle S$
  3.  $\triangle LMN \cong \triangle QRS$
1. Given
  2. 3rd  $\angle$  Thm
  3. ASA Post

Given:  $\angle DAC \cong \angle BEC$

$\overline{DC} \cong \overline{BC}$

Prove:  $\triangle ACD \cong \triangle ECB$

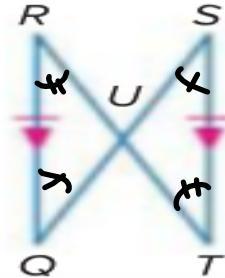


- 1.
  2.  $\angle C \cong \angle C$
  3.  $\triangle ACD \cong \triangle ECB$
1. Given
  2. Reflexive Prop
  3. AAS Thm.

Write a flow proof.

Given:  $\overline{RQ} \cong \overline{ST}$  and  $\overline{RQ} \parallel \overline{ST}$

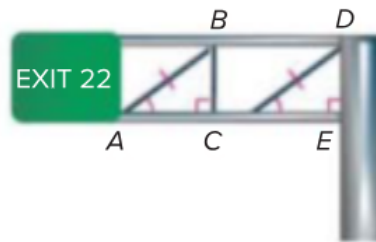
Prove:  $\triangle RUQ \cong \triangle TUS$



- |   |             |
|---|-------------|
| 1.  | 1. Given    |
| 2. $\angle Q \cong \angle S$<br>$\angle R \cong \angle T$ | 2. AIA Thm. |
| 3. $\triangle RUQ \cong \triangle TUS$                    | 3. ASA Post |

**Guided Practice**

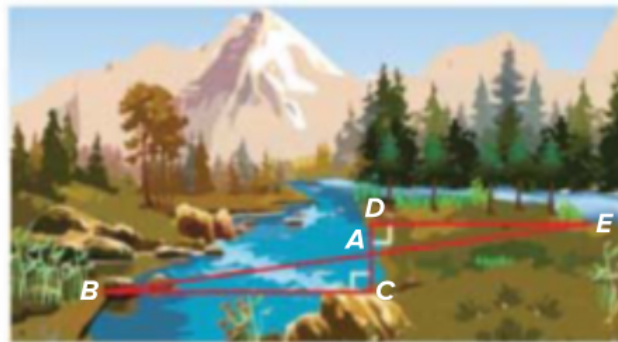
3. In the sign scaffold shown at the right,  $\overline{BC} \perp \overline{AC}$  and  $\overline{DE} \perp \overline{CE}$ .  $\angle BAC \cong \angle DCE$ , and  $\overline{AB} \cong \overline{CD}$ . Write a paragraph proof to show that  $\overline{BC} \cong \overline{DE}$ .








- |   |                                    |
|---|------------------------------------|
| 1.  | 1. Given                           |
| 2. $\angle C$ and $\angle E$ are rt $\angle$ 's | 2. Def of $\perp$ lines            |
| 3. $\angle C \cong \angle E$                    | 3. All rt. $\angle$ 's are $\cong$ |
| 4. $\triangle ABC \cong \triangle CDE$          | 4. AAS Thm.                        |
| 5. $\overline{BC} \cong \overline{DE}$          | 5. CPCTC                           |

**Real-World Example 3** Apply Triangle Congruence

**COMMUNITY SERVICE** Jeremias is working with a community service group to build a bridge across a creek at a local park. The bridge will span the creek between points  $C$  and  $B$ . Jeremias located a fixed point  $D$  to use as a reference point so that the segments have the relationships shown.  $A$  is the midpoint of  $\overline{CD}$  and  $DE$  is 15 feet. How long does the bridge need to be?

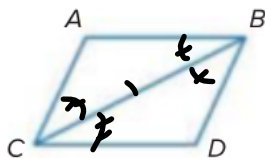


Concept Summary Proving Triangles Congruent 			
SSS	SAS	ASA	AAS
 <p>Three pairs of corresponding sides are congruent.</p>	 <p>Two pairs of corresponding sides and their included angles are congruent.</p>	 <p>Two pairs of corresponding angles and their included sides are congruent.</p>	 <p>Two pairs of corresponding angles and the corresponding nonincluded sides are congruent.</p>

1. two-column proof

**Given:**  $\overline{CB}$  bisects  $\angle ABD$  and  $\angle ACD$ .

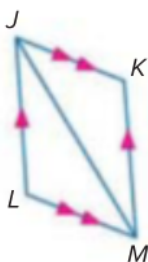
**Prove:**  $\triangle ABC \cong \triangle DCB$





**Given:**  $\overline{JK} \parallel \overline{LM}$ ,  $\overline{JL} \parallel \overline{KM}$

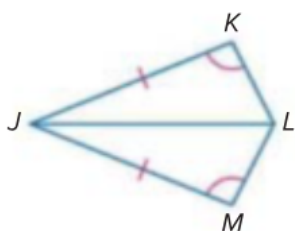
**Prove:**  $\triangle JML \cong \triangle MJK$



3. paragraph proof

**Given:**  $\angle K \cong \angle M$ ,  $\overline{JK} \cong \overline{JM}$ ,  
 $\overline{JL}$  bisects  $\angle KLM$ .

**Prove:**  $\triangle JKL \cong \triangle JML$



## 4. two-column proof

**Given:**  $\overline{GH} \parallel \overline{FJ}$ 

$$m\angle G = m\angle J = 90$$

**Prove:**  $\triangle HJF \cong \triangle FGH$ 