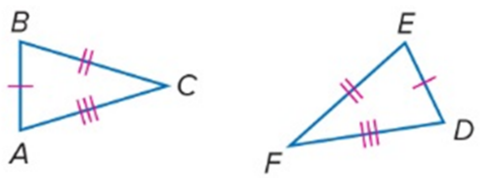
Section 4.3

Proving Triangles Congruent–SSS, SAS

Postulate 4.1 Side-Side-Side (SSS) Congruence

If three sides of one triangle are congruent to three sides of a second triangle, then the triangles are congruent.

Example If **S**ide $\overline{AB} \cong \overline{DE}$,
Side $\overline{BC} \cong \overline{EF}$, and
Side $\overline{AC} \cong \overline{DF}$,
then $\triangle ABC \cong \triangle DEF$.

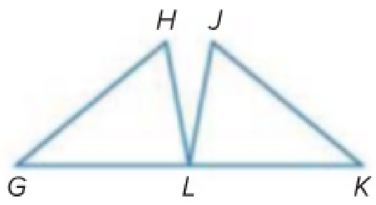


Example 1 Use SSS to Prove Triangles Congruent

Write a flow proof.

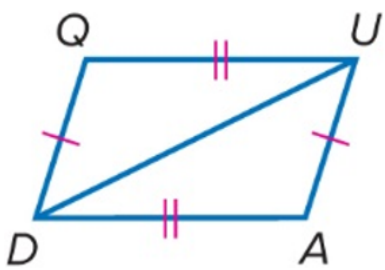
Given: $\overline{GH} \cong \overline{KJ}$, $\overline{HL} \cong \overline{JL}$, and L is the midpoint of $\overline{GK}$.

Prove: $\triangle GHL \cong \triangle KJL$



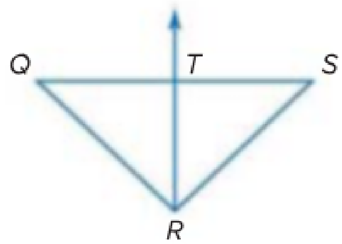
Given: $\overline{QU} \cong \overline{AD}$, $\overline{QD} \cong \overline{AU}$

Prove: $\Delta QUD \cong \Delta ADU$



Given: $\triangle QRS$ is isosceles with $\overline{QR} \cong \overline{SR}$.
$\overline{RT}$ bisects $\overline{QS}$ at point T.

Prove: $\triangle QRT \cong \triangle SRT$



Example 2 SSS on the Coordinate Plane

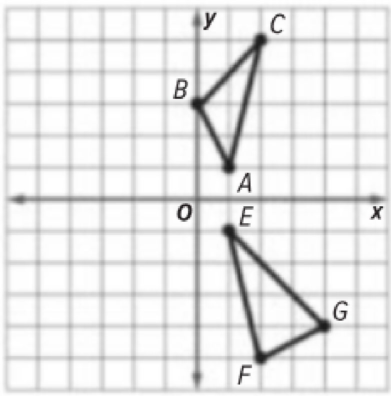
Triangle *ABC* has vertices *A*(1, 1), *B*(0, 3), and *C*(2, 5). Triangle *EFG* has vertices *E*(1, −1), *F*(2, −5), and *G*(4, −4).

a. Graph both triangles on the same coordinate plane.

b. Use your graph to make a conjecture as to whether the triangles are congruent. Explain your reasoning using rigid motions.

c. Write a logical argument using coordinate geometry to support the conjecture you made in part b.

a.

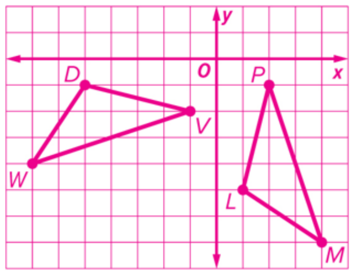


Triangle *DVW* has vertices *D* (–5, –1), *V* (–1, –2), and *W* (–7, –4). Triangle *LPM* has vertices *L* (1, –5), *P* (2, –1), and *M* (4, –7).

A. Graph both triangles on the same coordinate plane.

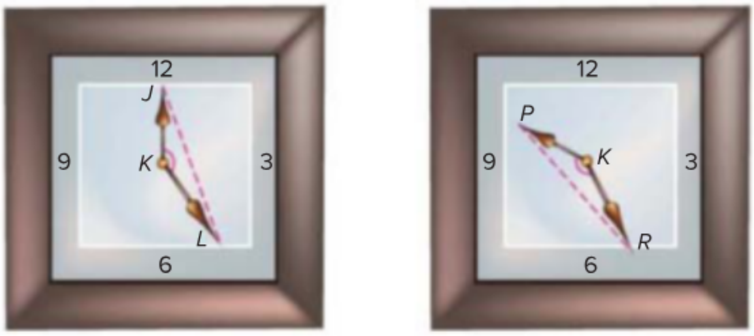
B. Use your graph to make a conjecture as to whether the triangles are congruent. Explain your reasoning using rigid motions.

C. Write a logical argument that uses coordinate geometry to support the conjecture you made in part B.



2 SAS Postulate

The angle formed by two adjacent sides of a polygon is called an **included angle**. Consider included angle *JKL* formed by the hands on the first clock shown below. Any time the hands form an angle with the same measure, the distance between the ends of the hands $\overline{JL}$ and $\overline{PR}$ will be the same.



$\triangle PKR \cong \triangle JKL$

Postulate 4.2 Side-Angle-Side (SAS) Congruence

Words If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then the triangles are congruent.

Example If **S**ide $\overline{AB} \cong \overline{DE}$,
Angle $\angle B \cong \angle E$, and
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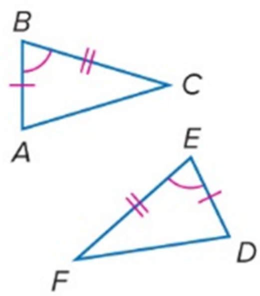
then $\triangle ABC \cong \triangle DEF$.



Postulate 4.2 Side-Angle-Side (SAS) Congruence

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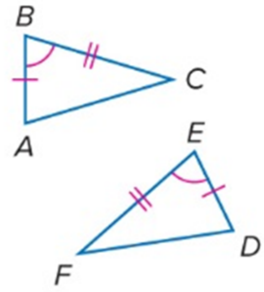
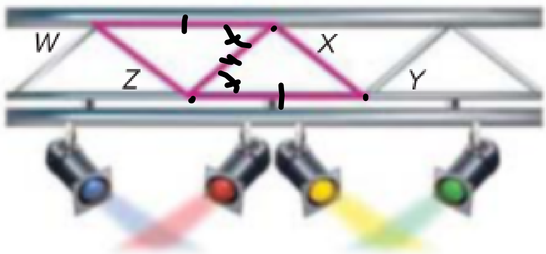
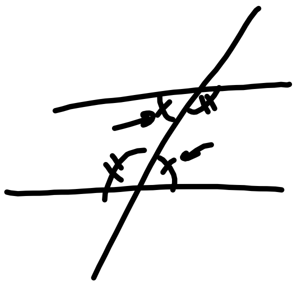


Real-World Example 3 Use SAS to Prove Triangles are Congruent

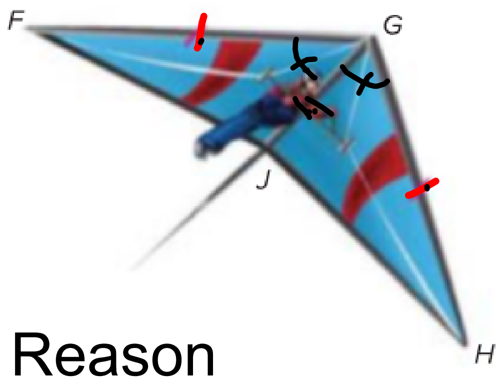
LIGHTING The scaffolding for stage lighting shown appears to be made up of congruent triangles. If $\overline{WX} \cong \overline{YZ}$ and $\overline{WX} \parallel \overline{ZY}$, write a two-column proof to prove that $\triangle WXZ \cong \triangle YZX$.



Statement	Reason
1.	1. Given
2. $\angle WXZ \cong \angle YZX$	2. AIA Thm
3. $\overline{XZ} \cong \overline{XZ}$	3. Reflective
4. $\triangle WXZ \cong \triangle YZX$	4. SAS Post

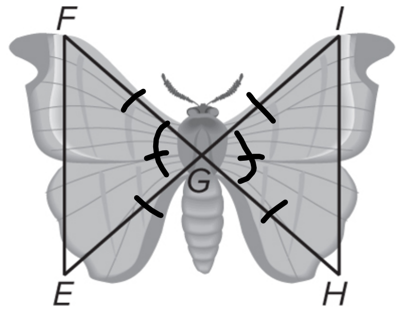


EXTREME SPORTS The wings of the hang glider shown appear to be congruent triangles. If $\overline{FG} \cong \overline{GH}$ and $\overline{JG}$ bisects $\angle FGH$, prove that $\triangle FGJ \cong \triangle HGJ$.



Statement	Reason
1.	1. Given
2. $\angle FGJ \cong \angle HGJ$	2. Def of $\angle$ bisector
3. $\overline{JG} \cong \overline{GJ}$	3. Reflective Prop
4. $\triangle FGH \cong \triangle HGJ$	4. SAS Post.

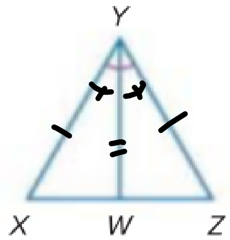
Entomology The wings of one type of moth form two triangles. Write a two-column proof to prove that $\triangle FEG \cong \triangle HIG$ if $\overline{EI} \cong \overline{FH}$, and G is the midpoint of both $\overline{EI}$ and $\overline{FH}$.



Statement	Reason
1.	1. Given
2. FG ≅ GH EG ≅ GI	2. Def of midpoint
3. ∠ FGE ≅ ∠HGI	3. Vertical ∠ Thm
4. ΔFGE ≅ Δ HGI	4. SAS Post.

Given: $\triangle XYZ$ is equilateral.
$\overline{WY}$ bisects $\angle XYZ$.

Prove: $\overline{XW} \cong \overline{ZW}$

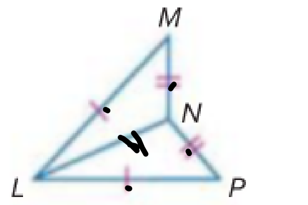


1.	1. Given
2. $\overline{XY} \cong \overline{YZ}$	2. Def of equilateral Δ
3. ∠XYW ≅ ∠ZYW	3. Def of ∠ bisector
4. $\overline{WY} \cong \overline{WY}$	4. Reflective
5. Δ XYW ≅ ΔZYW	5. SAS Post.
6 XW ≅ ZW	6. CPCTC

Write a two-column proof.

Given: $\overline{MN} \cong \overline{PN}$, $\overline{LM} \cong \overline{LP}$

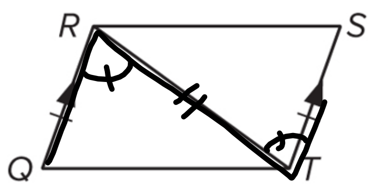
Prove: $\angle LNM \cong \angle LNP$

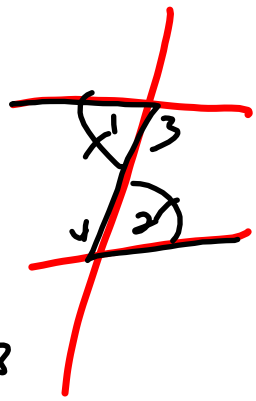


Statement	Reason
1	1. Given
2 LN ≅ LN	2. Reflective
3. △LNM = △LNP	3. SSS Post.
4. ∠LNM ≅ ∠LNP	4. CPCTC

Given: $\overline{RQ} \parallel \overline{TS}$, $\overline{RQ} \cong \overline{TS}$

Prove: $\angle Q \cong \angle S$

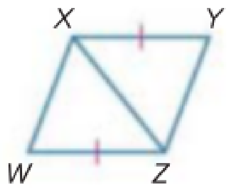




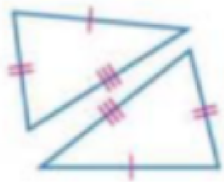
Statement	Reason
1.	1. Given
2. ∠QRT ≅ ∠STR	2. AIA Thm.
3. RT ≅ RT	3. Reflective Prop
4. △QRT ≅ △STR	4 SAS Post
5. ∠Q ≅ ∠S	5. CPCTC.

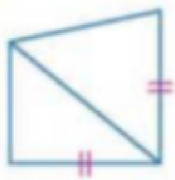
Given: $\overline{YX} \cong \overline{WZ}$, $\overline{YX} \parallel \overline{ZW}$

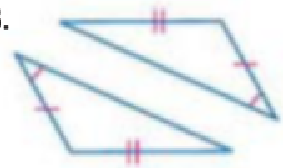
Prove: $\triangle YXZ \cong \triangle WZX$



CONSTRUCT ARGUMENTS Determine which postulate can be used to prove that the triangles are congruent. If it is not possible to prove congruence, write *not possible*. If it is possible, describe the rigid motions that map one triangle onto the other.

16. 

17. 

18. 

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