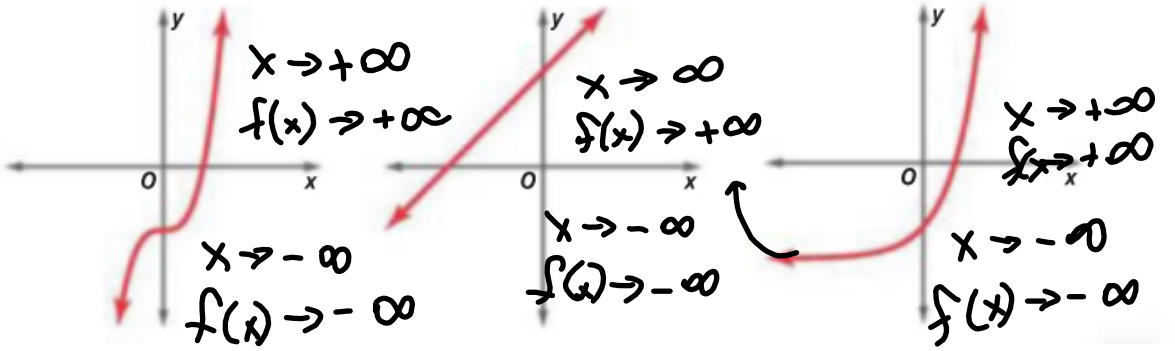
# Section 2.3 Extrema and End Behavior

## 1 End Behavior of Graphs of Functions

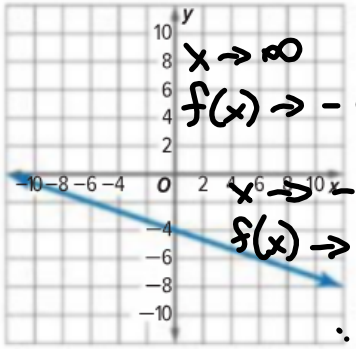
**End behavior** is the behavior of a graph as $x$ approaches positive or negative infinity. At the right end, the values of $x$ are increasing toward infinity. This is denoted as $x \to +\infty$. At the left end, the values of $x$ are decreasing toward negative infinity. This is denoted as $x \to -\infty$. For the graphs below, as $x \to +\infty$, $f(x)$ increases, so $f(x) \to +\infty$, and as $x \to -\infty$, $f(x)$ decreases, so $f(x) \to -\infty$. These graphs all have the same end behavior.



**Example 1** End Behavior of Linear Functions

Describe the end behavior of each linear function graph.

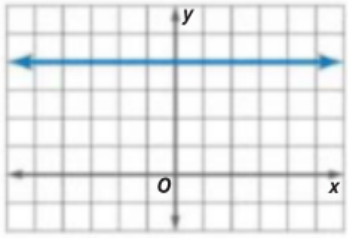
a.




$x \to \infty$, $f(x) \to -\infty$

$x \to -\infty$, $f(x) \to +\infty$

b.

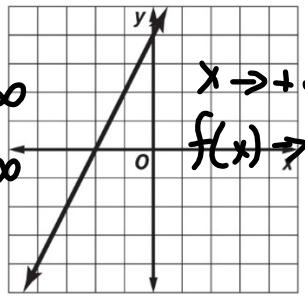



$x \to +\infty$, $f(x) \to 4$

$x \to -\infty$, $f(x) \to 4$

**Describe the end behavior of each linear function.**

A.

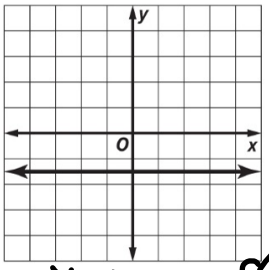



$x \to -\infty$, $f(x) \to -\infty$

$x \to +\infty$, $f(x) \to +\infty$

**Describe the end behavior of each linear function.**

B.



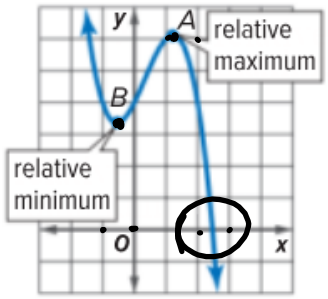

$y = -1.5$

$x \to +\infty$, $f(x) \to -1.5$

$x \to -\infty$, $f(x) \to -1.5$

## 2 Extrema of Functions

Sometimes the behavior of a function closer to the origin is important to know. The graph below shows a nonlinear function with coordinates close to the origin.




The point $A$ is a **relative maximum** of the function since no other nearby points have a greater $y$-coordinate. The graph is increasing as it approaches $A$ and decreasing as it moves away from $A$.

The point $B$ is a **relative minimum** of the function since no other nearby points have a lesser $y$-coordinate. The graph is decreasing as it approaches $B$ and increasing as it moves away from $B$.

The relative maxima and relative minima are often referred to as **turning points**, or the points where a curve completely changes direction from either down to up or up to down.

mini 0 to -1

max 1 to 2

X-Coor zeros 2 to 3

The relative minima, relative maxima, and turning points are known as the **extrema** of the graph.

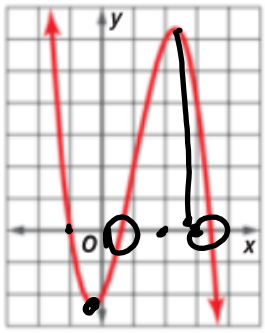
The table and graph below are of a function with extrema. Estimate the zeros. Then estimate the values of $x$ at which relative maxima and minima occur.

zeros = where the graph crosses the x-axis

## Guided Practice

3.

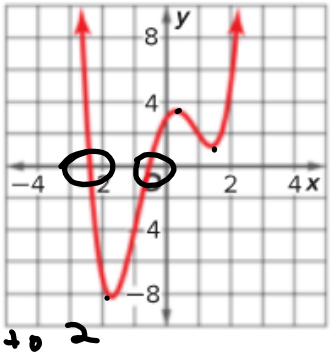
| $x$ | $f(x)$ |
|---|---|
| −2 | 14 |
| −1 | 0 |
| 0 | −2 |
| 1 | 2 |
| 2 | 6 |
| 3 | 4 |
| 4 | −10 |




min 0 to −1
max 2 to 3
zero −1, 0 to 1, 3 to 4

The table and graph below are of a function with extrema. Estimate the zeros. Then estimate the values of $x$ at which relative maxima and minima occur.

| $x$ | $f(x)$ |
|---|---|
| −3 | 30 |
| −2 | 7 |
| −1 | −4 |
| 0 | 3 |
| 1 | 2 |
| 2 | 5 |
| 3 | 48 |




mini −1 to −2, 1 to 2

max 0 to 1

zero's −2 to −3, 0 to −1

←0

mini 0 to 1

max 0 to −1

zero −2

| $x$ | $f(x)$ |
|---|---|
| −3 | −18 |
| −2 | −1 |
| −1 | 4 |
| 0 | 3 |
| 1 | 2 |
| 2 | 7 |
| 3 | 24 |