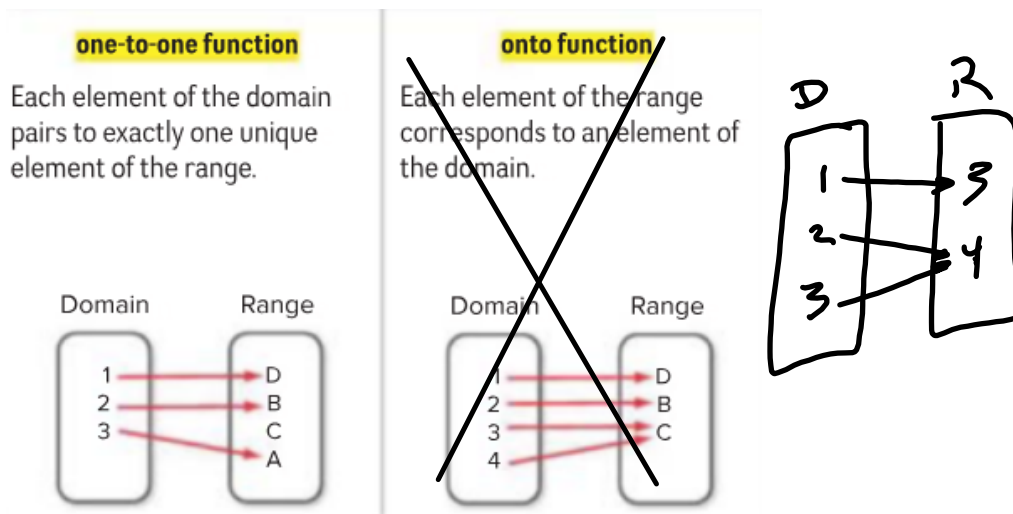Section 2.1 Functions and Continuity

A **_function_** is a relations in which each element in the domain is paired with exactly one element in the range.

Mapping Functions

one-to-one function

Each element of the domain pairs to exactly one unique element of the range.

onto function

Each element of the range corresponds to an element of the domain.

Domain   Range
1 → D
2 → B
3 → C
→ A

Domain   Range
1 → D
2 → B
3 → C
4 → C

D R

1 → 3
2 → 4
3

Example 1   Domain and Range

State the domain and range of each relation. Then determine whether each relation is a *function*. If it is a function, determine if it is *one-to-one, ~~onto, both, or neither~~*.

a. $\{(-6, -1), (-5, -9), (-3, -7), (-1, 7), (6, -9)\}$

D: $\{-6, -5, -3, -1, 6\}$ Function
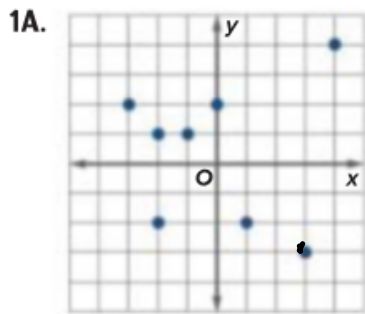
R: $\{-1, -9, -7, 7\}$ NOT 1 to 1

b.

x	2	−1	−2	−1	2
y	−2	−1	0	1	2

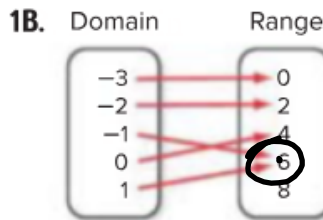Domain: $\{-2, -1, 2\}$ Range: $\{-2, -1, 0, 1, 2\}$

NOT a function
NOT 1 to 1

State the domain and range of each relation. Then determine whether each relation is a *function*. If it is a function, determine if it is *one-to-one, ~~onto, both, or neither~~*.
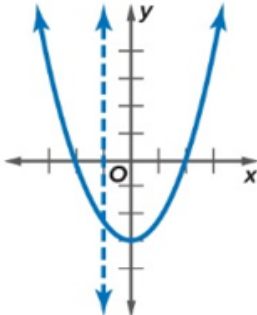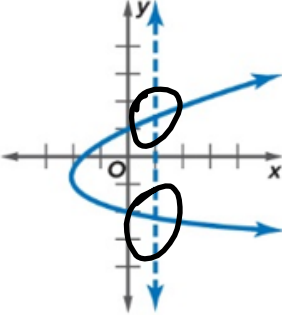
1A.   1B.

NOT function   function
NOT 1 to 1

D: $\{-3, -2, -1, 0, 1, 3, 4\}$

R: $\{2, 1, -2, 1, -3, 4\}$

The Vertical Line Test

Key Concept Vertical Line Test

| Words | If no vertical line intersects a graph in more than one point, the graph represents a function. | If a vertical line intersects a graph in two or more points, the graph does not represent a function. |

Models

Example 2   Graph a Relation

Graph $y = \frac{1}{2}x - 3$, and determine the domain and range. Then determine whether the equation is a *function*, is *one-to-one*, ~~onto, both, or neither~~. State whether it is *discrete* or *continuous*.

$$D: \{ x \mid x \text{ is all } \mathbb{R} \}$$

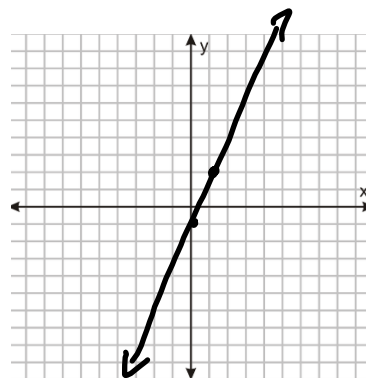$$R: \{ y \mid y \text{ is all } \mathbb{R} \}$$

func
1 to 1
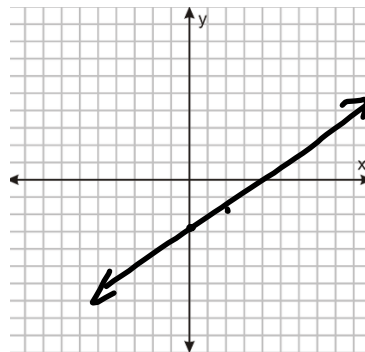
Graph y = 3x – 1 and determine the domain and range. Then determine whether the equation is a *function*, is *one-to-one*, ~~onto, both, or neither~~. State

$$D: \{ x \mid x \text{ is all } \mathbb{R} \}$$

$$R: \{ y \mid y \text{ is all } \mathbb{R} \}$$

function
1 to 1

When an equation represents a function, the variable, often x, with values making up the domain is called the **independent variable**. The other variable, often y, is called the **dependent variable** because its values depend on x.

Equations that represent functions are often written in **function notation**. The equation $y = 5x - 1$ can be written as $f(x) = 5x - 1$.

Reading Math

MP **Structure** The symbol $f(x)$ replaces the y and is read "f of x." The f is just the name of the function. It is not a variable that is multiplied by x.

$$h(x) = 5x - 1$$

$$g(x) = 5x - 1$$

$$n(x) = 5x - 1$$

Example 3   Evaluate a Function

Given $f(x) = 2x^2 - 8$, find each value.

a. $f(6) = 2(6)^2 - 8$

$= 2(36) - 8$

$= 72 - 8$

$= 64 \quad (6, 64)$

b. $f(2y) = 2(2y)^2 - 8$

$= 2(4y^2) - 8$

$= 8y^2 - 8$

$\left(2y, 8y^2 - 8\right)$

Given $f(x) = x^3 - 3$, find each value.

A. $f(-2) = (-2)^3 - 3$

$= -8 - 3$

$= -11$

$(-2, -11)$

B. $f(2t) = (2t)^3 - 3$

$= 8t^3 - 3$

$\left(2t, 8t^3 - 3\right)$

Given $g(x) = 0.5x^2 - 5x + 3.5$, find each value.

3A. $g(2.8) = .5(2.8)^2 - 5(2.8) + 3.5$

$= -6.58$

$(2.8, -6.58)$

3B. $g(4a) = .5(4a)^2 - 5(4a) + 3.5$

$= 8a^2 - 20a + 3.5$

MP **STRUCTURE** State the domain and range of each relation. Then determine whether each relation is a *function*. If it is a function, determine if it is *one-to-one, ~~onto, both, or neither~~*.

1.

5 → 3
6 → 1
−2 → −8

2.

(−2, 3)
(1, 5)
(1, 2)
(4, −1)
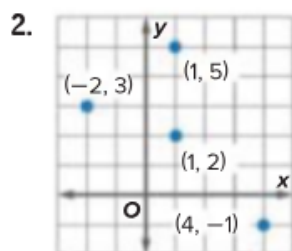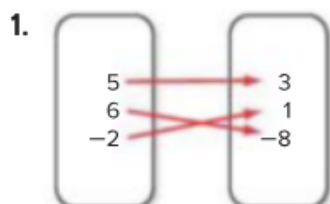
3

x	y
−2	−4
1	−4
4	−2
8	6

4. $\{(-3, 4), (2, -1), (-2, -1), (6, 2), (5, 4)\}$

Evaluate each function.

10. $f(-3)$ if $f(x) = -4x - 8$

11. $g(5)$ if $g(x) = -2x^2 - 4x + 1$

Find each value if $f(x) = 3x + 2$, $g(x) = -2x^2$, and $h(x) = -4x^2 - 2x + 5$.

28. $f(-5)$ **29.** $f(9)$ **30.** $g(-3)$
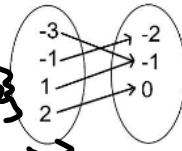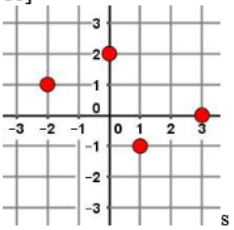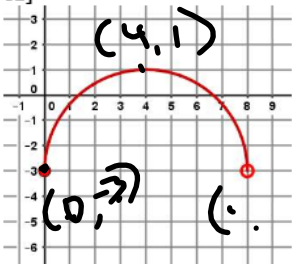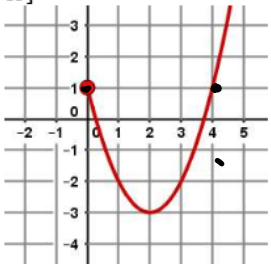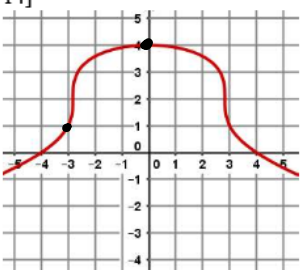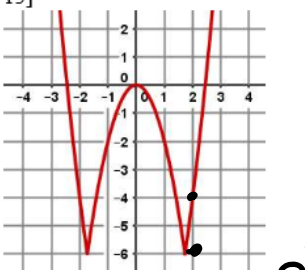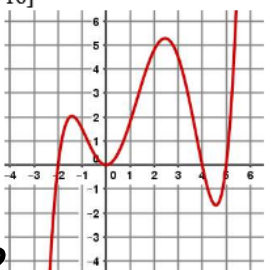
31. $g(-6)$ **32.** $h(3)$ **33.** $h(8)$

Identify the domain and range, then evaluate each function for the given value of x.

D   R

8] $f = \{(10,7),(-2,4),(5,3),(4,10)\}$	9]	10]
Domain: $\{10,-2,5,4\}$	Domain:	Domain: $\{-3,-1,1\}$
Range: $\{7,4,3,10\}$	Range: $\{3,-1,0,1\}$ Range: $\{3,1,2,1\}$	Range $\{-2,-1,0\}$
$f(10) = 7$ $(10,7)$	$f(-1) = 1$	$f(-3) = $ —

In #9:

x	y
-3	3
-1	1
0	0
1	1

In #10 mapping: -3, -1, 1, 2 → -2, -1, 0

11]	12]	13]
	(4,1)	
	$(0,\overset{?}{3})$ $(\cdot,\cdot)$	
Domain:	Domain: $\{x \mid 0 \leq x < 8\}$	Domain: $\{x \mid x \geq 0\}$
Range:	Range: $\{y \mid -3 \leq y \leq 1\}$	Range: $\{y \mid y \geq -3\}$
$f(3) = $	$f(0) = -3$	$f(4) = 1$

$\leq$ $\leq$

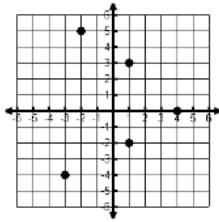| 14]	15]	16]
Domain: $\{x \mid x \text{ is all } \mathbb{R}\}$	Domain: $\{x \mid x \text{ is all } \mathbb{R}\}$	Domain:
Range: $\{y \mid y \leq 4\}$	Range: $\{y \mid y \geq -6\}$	Range:
$f(-3) = 1$	$f(2) = -4$	$f(-2) = $

Domain and Range Notes   NAME:_____
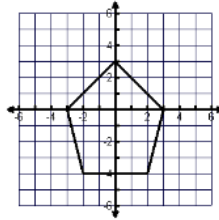
State the domain and range for each graph and then tell if the graph is a function (write yes or no).
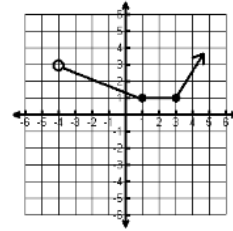If the graph is a function, state whether it is discrete, continuous or neither.

1) Domain_____
    Range_____
    Function?_____

2) Domain_____
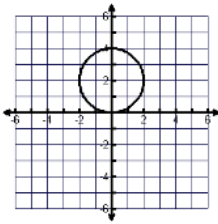   Range_____
    Function?_____

3) Domain_____
    Range_____
    Function?_____

4) Domain_____
    Range_____
    Function?_____

5) Domain_____
    Range_____
    Function?_____
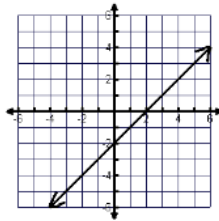
6) Domain_____
    Range_____
    Function?_____
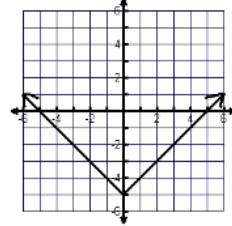
7) Domain_____
    Range_____
    Function?_____
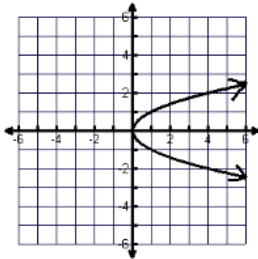
8) Domain_____
    Range_____
    Function?_____

9) Domain_____
    Range_____
    Function?_____