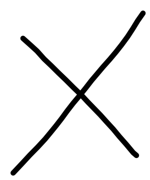
If a rock and a feather are dropped from the same height in a vacuum chamber, the two objects fall at the same rate. This demonstrates one of Sir Isaac Newton's laws of gravity and inertia. These laws are accepted as fundamental truths of physics. Some laws in geometry must also be accepted as true.

Why do a rock and a feather not fall at the same rate normally?

Section 2.4 Writing Proofs

Postulates: A postulate, is a statement that is accepted as true without proof. Basic ideas about points, lines, and planes can be stated as postulates.

Postulate 2.1: Through any two points, there is exactly one line.



Postulate 2.2: Through any three non-collinear points, there is exactly one plane.

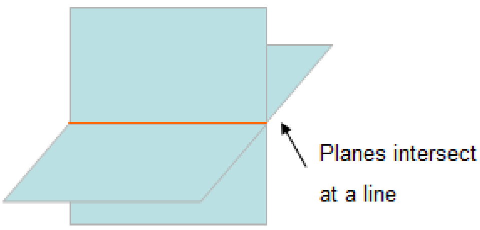
Postulate 2.3: A line contains at least two points

Postulate 2.4: A plane contains at least three non-collinear points.

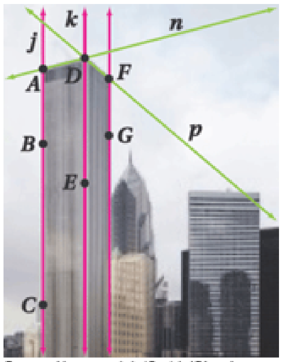
Postulate 2.5: If two points lie in a plane, then the entire line containing those points lies in that plane.

Postulate 2.6: If two lines intersect, then their intersection is exactly one point.

Postulate 2.7: If two planes intersect, then their intersection is a line.



Example: Explain how the photo illustrates that each statement is true. Then state the postulate that can be used to show each statement is true.



George Hammerstein/Corbis/Glow Images

a. Lines *n* and *p* intersect at point D.

b. Points A, B, and D form a plane.

c. Points A, B and C are points that make up line *j*.

d. the plane that contains A, D and E intersects the plane that contains F, G, and E in line *k*

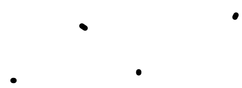
Example: Determine whether each statement is always, sometimes, or never true. Explain your reasoning.

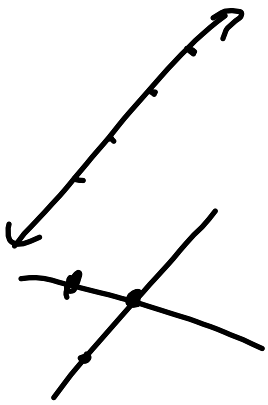
a. If two coplanar lines intersect, then the point of intersection lies in the same plane as the two lines.

Always Post 2.5

b. Four points are non-collinear.

Sometimes





c. Two intersecting lines determine a plane.

Alway 2.2

d. Three lines intersect in two points. Never x - 3 = 6

Key Concept Properties of Real Numbers

The following properties are true for any real numbers *a*, *b*, and *c*.

Addition Property of Equality	If $a = b$, then $a + c = b + c$.
Subtraction Property of Equality	If $a = b$, then $a - c = b - c$.
Multiplication Property of Equality	If $a = b$, then $a \cdot c = b \cdot c$.
Division Property of Equality	If $a = b$ and $c \neq 0$, then, $\frac{a}{c} = \frac{b}{c}$.
Reflexive Property of Equality	$a = a$
Symmetric Property of Equality	If $a = b$, then $b = a$.
Transitive Property of Equality	If $a = b$ and $b = c$, then $a = c$.
Substitution Property of Equality	If $a = b$, then *a* may be replaced by *b* in any equation or expression.
Distributive Property of Equality	$a(b + c) = ab + ac$

x + 4 = 2

$\frac{2x}{2} = 3 \cdot 4$

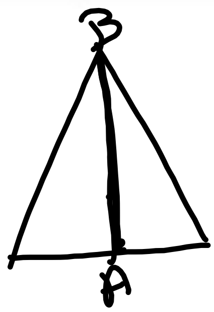
$\frac{2x}{2} = \frac{8}{2}$

4 = 3 3 = 4

x + 2 = 4

4 = x + 2

2x + 3 when x = 4



Proofs: To prove a conjecture, you use deductive reasoning to move from a hypothesis to the conclusion of the conjecture you are trying to prove. This done by writing a proof which is a logical argument in which each statement you make is supported by a statement that is accepted as true. (Postulates and definitions)

How to write a two column proof

1. Write the given statement in the first column and the reason in the second column.

2. Create a deductive argument by forming a logical chain of statements linking the given to what you are trying to prove. Write the statements in the first column and the justification in the second column.

3. State what it is that you have proved.

Prove that if $\frac{y+2}{3} = 3$,then y=7

Statement	Justification
1. $\frac{y+2}{3} = 3$	1. Given
2. $3 \cdot \frac{y+2}{3} = 3 \cdot 3$	2. Mult. Prop
3 $y+2 = 9$	3. Substitution
4. $y+2-2 = 9-2$	4. Subtraction Prop
5 $y = 7$	5. Substitution

Prove that if -5(x+4)=70) , then x=18

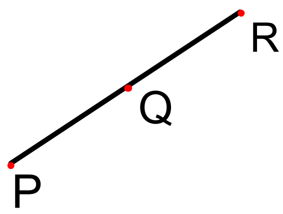
Statement	Justification
1. $-5(x+4)=70$	1. Given
2. $-5x + -5(4) = 70$	2. Distribute
3. $-5x - 20 = 70$	3. Substitution Prop
4. $-5x - 20 + 20 = 70 + 20$	4. Add Prop.
5. $-5x = 90$	5. Substitution
6. $\frac{-5}{-5}x = \frac{90}{-5}$	6. Division
7. $x = -18$	7. Sub.

Once a statement or conjecture has been proved, it is called a theorem, and it can be used as a reason to justify statements in other proofs.

Example: Write a two column proof.

Given: Q is the midpoint of PR

Prove: PQ is congruent to QR

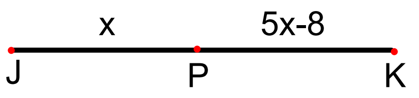


1. Q is midpt of $\overline{PR}$	1. Given
2. $PQ = QR$	2. Def of midpt
3. $\overline{PQ} \cong \overline{QR}$	3. Def of $\cong$.

Example of a proof.

Given that P is the midpoint of JK, prove that x = 2

What is the definition of Midpoint?



JP=PK

Substitute in for each segment.

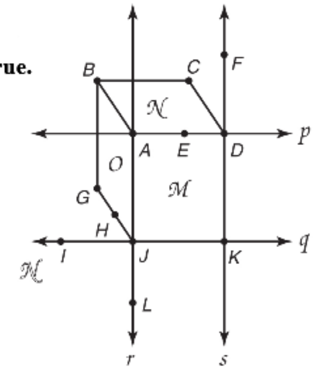
The example above is known as the Midpoint Theorem.

The Midpoint Theorem: If M is the midpoint of AB, the AM ≅ MB

NAME ________________________________ DATE ____________________ PERIOD __________

2-4 Skills Practice
Writing Proofs

Explain how the figure illustrates that each statement is true. Then state the postulate that can be used to show each statement is true.



1. Planes $\mathcal{O}$ and $\mathcal{M}$ intersect in line r.

2. Line p lies in plane $\mathcal{N}$.

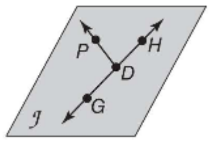
Determine whether each statement is *always*, *sometimes*, or *never* true. Explain your reasoning.

3. Three collinear points determine a plane.

4. Two points A and B determine a line.

5. A plane contains at least three lines.

In the figure, $\overleftrightarrow{DG}$ and $\overleftrightarrow{DP}$ are in plane $\mathcal{J}$ and H lies on $\overleftrightarrow{DG}$. State the postulate that can be used to show each statement is true.



6. G and P are collinear.

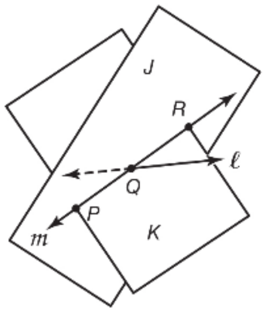
7. Points D, H, and P are coplanar.

8. PROOF In the figure at the right, point B is the midpoint of $\overleftrightarrow{AC}$ and point C is the midpoint of $\overleftrightarrow{BD}$. Write a paragraph proof to prove that $AB = CD$.

NAME ______________________________ DATE ____________________ PERIOD ________

2-4 Practice
Writing Proofs

Explain how the figure illustrates that each statement is true. Then state the postulate that can be used to show each statement is true.



1. The planes J and K intersect at line m.

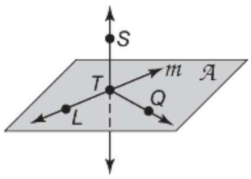
2. The lines ℓ and m intersect at point Q.

Determine whether the following statements are *always*, *sometimes*, or *never* true. Explain.

3. The intersection of two planes contains at least two points.

4. If three planes have a point in common, then they have a whole line in common.

In the figure, line m and $\overrightarrow{TQ}$ lie in plane $\mathcal{A}$. State the postulate that can be used to show that each statement is true.



5. Points L, and T and line m lie in the same plane.

6. Line m and $\overleftrightarrow{ST}$ intersect at T.

7. In the figure, E is the midpoint of $\overline{AB}$ and $\overline{CD}$, and $AB = CD$.
Write a two column proof to prove that $\overline{AE} \cong \overline{ED}$.