Section 2.1 Relations and Functions

The table shows the average lifetime and maximum lifetime for some animals. The data can also be represented as **ordered pairs**. The ordered pairs for the data are (12, 28), (15, 30), (8, 20), (12, 20), and (20, 50). The first number in each ordered pair is the average lifetime, and the second number is the maximum lifetime.

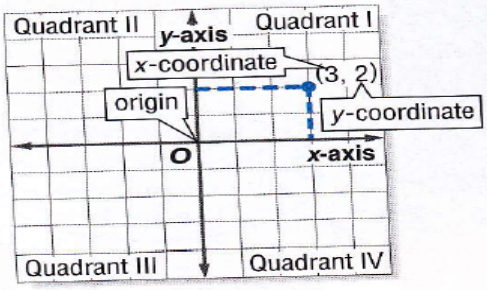
Animal	Average Lifetime (years)	Maximum Lifetime (years)
Cat	12	28
Cow	15	30
Deer	8	20
Dog	12	20
Horse	20	50

(12, 28)

average lifetime ↑ ↑ maximum lifetime

In general, any ordered pair in the coordinate plane can be written in the form (x, y).

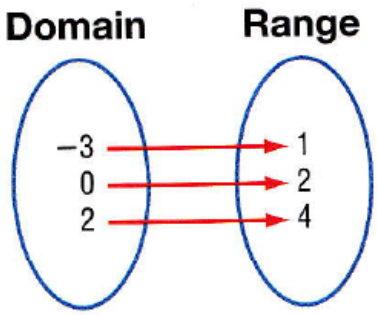
A **relation** is a set of ordered pairs, such as the one for the longevity of animals. The **domain** of a relation is the set of all first coordinates (x-coordinates) from the ordered pairs, and the **range** is the set of all second coordinates (y-coordinates) from the ordered pairs. The *graph* of a relation is the set of points in the coordinate plane corresponding to the ordered pairs in the relation.



Assume that each square on a graph represents 1 unit unless otherwise labeled.

A **function** is a special type of relation in which each element of the domain is paired with *exactly one* element of the range. A **mapping** shows how each member of the domain is paired with each member of the range.

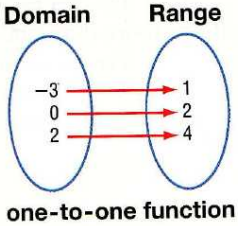
{(−3, 1), (0, 2), (2, 4)}



The first two relations shown below are functions. The third relation is not a function because the −3 in the domain is paired with both 0 and 6 in the range. A function like the first one below, where each element of the range is paired with exactly one element of the domain, is called a **one-to-one function**.

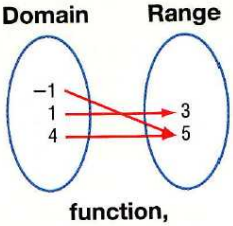
Concept Summary *Functions*

{(−3, 1), (0, 2), (2, 4)}



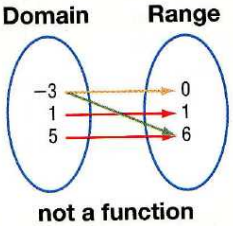
one-to-one function

{(−1, 5), (1, 3), (4, 5)}



function, not one-to-one

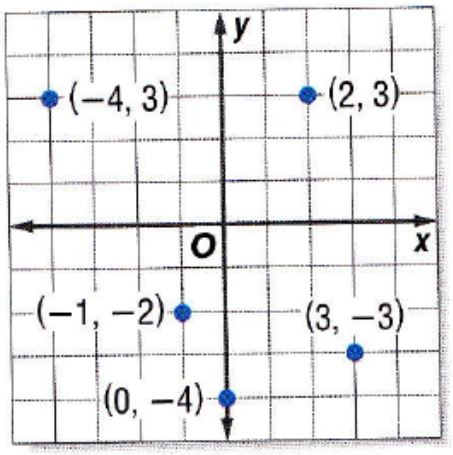
{(5, 6), (−3, 0), (1, 1), (−3, 6)}



not a function

Example 1 Domain and Range

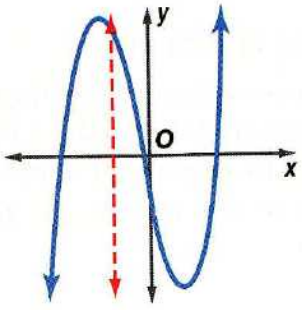
State the domain and range of the re
graph. Is the relation a function?

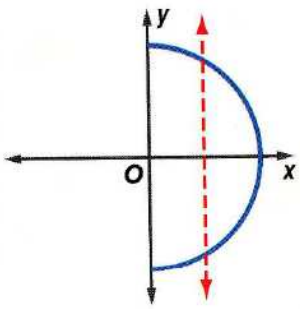


You can use the **vertical line test** to determine whether a relation is a function.

Key Concept Vertical Line Test

- **Words** If no vertical line intersects a graph in more than one point, the graph represents a function. If some vertical line intersects a graph in two or more points, the graph does not represent a function.
- **Models**



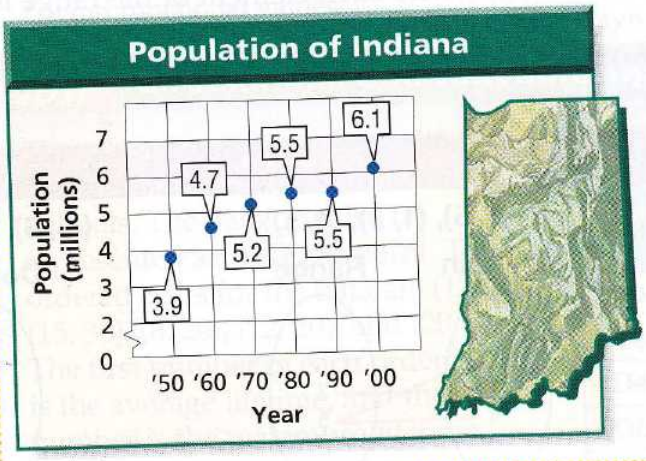


Example 2 Vertical Line Test

GEOGRAPHY The table shows the population of the state of Indiana over the last several decades. Graph this information and determine whether it represents a function.

Year	Population (millions)
1950	3.9
1960	4.7
1970	5.2
1980	5.5
1990	5.5
2000	6.1

Source: U.S. Census Bureau



Use the vertical line test. Notice that no vertical line can be drawn that contains more than one of the data points. Therefore, this relation is a function. *Notice also that each year is paired with only one population value.*

Example 3 Graph Is a Line

a. **Graph the relation represented by $y = 2x + 1$.**

Make a table of values to find ordered pairs that satisfy the equation. Choose values for x and find the corresponding values for y. Then graph the ordered pairs.

b. Find the Domain and Range

c. Is the relation a function?

Example 4 Graph Is a Curve

a. Graph the relation represented by $x = y^2 - 2$.

Make a table. In this case, it is easier to choose y values and then find the corresponding values for x. Then sketch the graph, connecting the points with a smooth curve.

x	y
	−2
	−1
	0
	1
	2

b. Find the Domain and Range

c. Is the relation a function?

When an equation represents a function, the variable, usually x, whose values make up the domain is called the **independent variable**. The other variable, usually y, is called the **dependent variable** because its values depend on x.

functional notation

$y = 2x + 1$ can be written as $f(x) = 2x + 1$.

Example 5 Evaluate a Function

Given $f(x) = x^2 + 2$ and $g(x) = 0.5x^2 - 5x + 3.5$, find each value.

a. $f(-3)$

b. $g(2.8)$

c. $f(3z)$

Practice and Apply

Homework Help

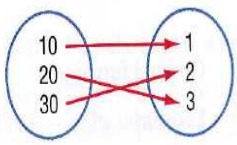
For Exercises	See Examples
17–28	1, 2
29–32	3
33, 34	4
35–45, 55	2
46–54, 56	5

Extra Practice

See page 830.

Determine whether each relation is a function. Write *yes* or *no*.

17.



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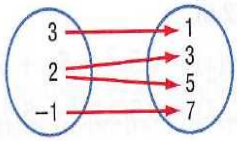


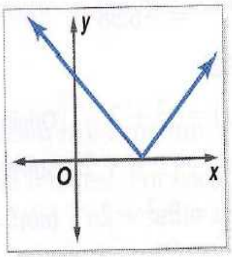
19.

x	y
0.5	−3
2	0.8
0.5	8

20.

x	y
2000	$4000
2001	$4300
2002	$4000
2003	$4500

21.



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