27. *License Plate* A license plate is to have five uppercase letters or digits. Determine the number of license plates possible if repetition is permitted and if any position can contain either a letter or digit with the exception that the first position cannot contain the letter O or the number 0.

30. ***Geometric Shapes*** Consider the five figures shown.

In how many different ways can the figures be arranged

a) from left to right? $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

b) from top to bottom if placed one under the other? 120

c) from left to right if the triangle is to be placed on the far right? $4 \cdot 3 \cdot 2 \cdot 1 \cdot 1 = 24$

d) from left to right if the circle is to be placed on the far left and the triangle is to be placed on the far right?

$1 \cdot 3 \cdot 2 \cdot 1 \cdot 1 = 6$

31. ***Arranging Pictures*** The six pictures shown are to be placed side by side along a wall.

In how many ways can they be arranged from left to right if

a) they can be arranged in any order? $6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$

b) the bird must be on the far left? $1 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

c) the bird must be on the far left and the giraffe must be next to the bird? $1 \cdot 1 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 24$

d) a four-legged animal must be on the far right? $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 5 = 600$

A **Permutation** is any ordered arrangement of a given set of objects.


Curly Moe Larry

example:

Larry, Moe, Curly Curly, Larry, Moe

The same three names yet two different ordered arrangements (permutaion)

How many different arrangements (permutations) of three object.

$3! = 3 \cdot 2 \cdot 1 = 6$ $3 \cdot 2 \cdot 1$ $3!$

When determining the number of permutations possible, we assume that repetition of an item is not permitted.

Example: Larry, Moe, Moe

Example 3

In how many ways can 7 children be arranged in a line?

$7! = 5040$

Example 4

Consider the five letters a, b, c, d, e. In how many distinct ways can three letters be selected and arranged if repetion is not allowed?

$5 \cdot 4 \cdot 3 = 60$

abc
abd
abe
acd

we just determined the number of different ways in which we could select and arrange three of the five items.

It is written ${}_5P_3$

This is read "the number of permutions of five items taken three at a time"

In general a permution when selecting fewer items then give is written

${}_nP_r$

Example

How many different arrangements or permutaions?

$_8P_3$ $8 \cdot 7 \cdot 6 = 336$

$_9P_4$ $9 \cdot 8 \cdot 7 \cdot 6 = 3024$

$_6P_2$ $6 \cdot 5 = 30$

Example 6

The Prince George County bicycle club has 10 different routes members wish to travel exactly once, but they only have 6 specific dates for their trips. In how many ways can the different routes be assigned to the dates scheduled for their trips?

$_{10}P_6 = 151,200$

$_{52}P_5$ 311,875,200

Permutation of Duplicate Items

How many permutions of the letters in the name BOB are possible?

If the two Bs are distinguishable (one red and on blue), there would be six permutations.

BOB

The number of distinct permutations of n objects where n_1 of the objects are identical, n_2 of the objects are identical ... n_r of the objects are identical is found by the formula

$$\frac{n!}{n_1! \cdot n_2! \cdot n_r!}$$

Example 7

In how many different ways can the letters of the word "TALLAHASSEE" be arranged?

$$\frac{11!}{3! \cdot 2! \cdot 2! \cdot 2!}$$

$6 \cdot 2 \cdot 2 \cdot 2 = 48$

83160

$n = 11$

$n_1 = A = 3$

$n_2 = L = 2$

$n_3 = S = 2$

$n_4 = E = 2$

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53. Determine the number of permutations of the letters c word "EDUCATION." $\frac{9!}{} =$ 362880

54. Determine the number of permutations of the letters c word "SELECTION." $\frac{9!}{2!} = 181{,}440$

55. In how many ways can the letters in the word "DIFFERENCE" be arranged?

56. In how many ways can the letters in the word "MISSISSIPPI" be arranged?

Letter Codes *In Exercises 37–40, an identification code is to consist of three letters followed by four digits. How many different codes are possible if*

37. repetition is not permitted?

38. repetition is permitted?

39. repetition of letters is permitted, repetition of numbers is not permitted, and the first three entries must all be the same letter?

40. the first letter must be an A, B, C, or D and repetition is not permitted?

License Plates *In Exercises 41–44, a license plate is to consist of three digits followed by two uppercase letters. Determine the number of different license plates possible if*

41. repetition of numbers and letters is permitted.

42. repetition of numbers and letters is not permitted.

43. the first and second digits must be odd, and repetition is not permitted.

44. the first digit cannot be zero, and repetition is not permitted.

45. ***Possible Phone Numbers*** A telephone number consists of seven digits with the restriction that the first digit cannot be 0 or 1.

a) How many distinct telephone numbers are possible?

b) How many distinct telephone numbers are possible with three-digit area codes preceding the seven-digit number, where the first digit of the area code is not 0 or 1?

c) With the increasing use of cell phones and paging systems, our society is beginning to run out of usable phone numbers. Various phone companies are developing phone numbers that use 11 digits instead of 7. How many distinct phone numbers can be made with 11 digits assuming that the area code remains three digits and the first digit of the area code and the phone number cannot be 0 or 1?

46. ***Granola Bars*** Mrs. Williams and her four children go shopping at a local grocery store. Each of the children will be allowed to select one box of granola bars. On the store's shelf there are 12 boxes of granola bars, and each box contains a different type of bar. In how many ways can the selections be made?