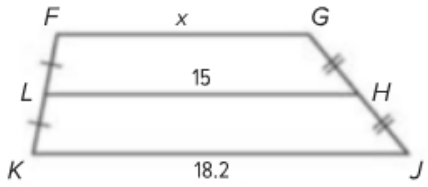
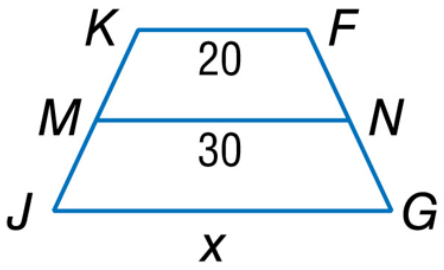
Example 3 Midsegment of a Trapezoid

In the figure, $\overline{LH}$ is the midsegment of trapezoid $FGJK$. What is the value of x?

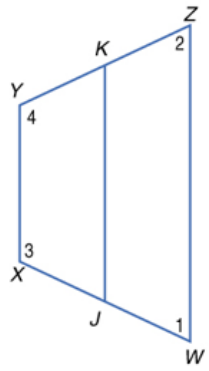


Note: The figure is not drawn to scale.

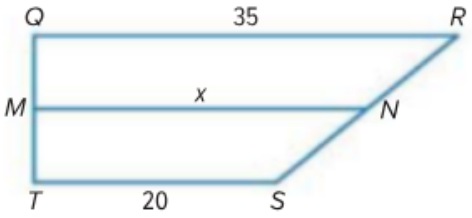
In the figure, $\overline{MN}$ is the midsegment of trapezoid $FGJK$. What is the value of x?



WXYZ is an isosceles trapezoid with midseg $\overline{JK}$. Find _XY_ if _JK_ = 18 and _WZ_ = 25.

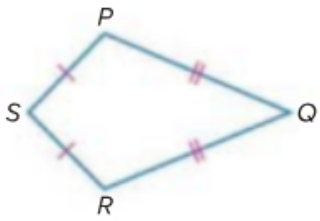


In the figure, $\overline{MN}$ is the midpoint of trapezoid $QRST$. Write the equation for the midpoint, and then solve for the value of x.



2 Properties of Kites

A **kite** is a quadrilateral with exactly two pairs of consecutive congruent sides. Unlike a parallelogram, the opposite sides of a kite are not congruent or parallel.



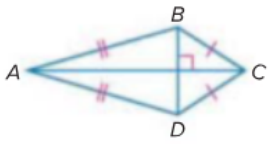
Study Tip

Kites The congruent angles of a kite are included by the noncongruent adjacent sides.

Theorems Kites

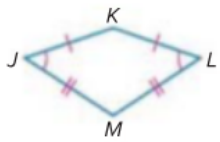
6.25 If a quadrilateral is a kite, then its diagonals are perpendicular.

Example If quadrilateral $ABCD$ is a kite, then $\overline{AC} \perp \overline{BD}$.



6.26 If a quadrilateral is a kite, then exactly one pair of opposite angles is congruent.

Example If quadrilateral $JKLM$ is a kite, $\overline{JK} \cong \overline{KL}$, and $\overline{JM} \cong \overline{LM}$, then $\angle J \cong \angle L$ and $\angle K \ncong \angle M$.



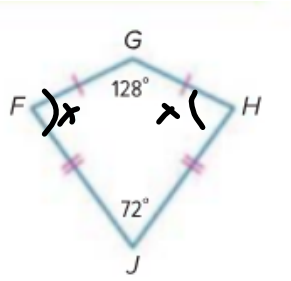
You will prove Theorems 6.25 and 6.26 in Exercises 31 and 32, respectively.

You can use the theorems above, the Pythagorean Theorem, and the Polygon Interior Angles Sum Theorem to find missing measures in kites.

Example 4 Use Properties of Kites

a. If *FGHJ* is a kite, find $m\angle GFJ$.

Because a kite can only have one pair of opposite congruent angles and $\angle G \ncong \angle J$, then $\angle F \cong \angle H$. So, $m\angle F = m\angle H$. Write and solve an equation to find $m\angle F$.



$128 + 72 + 2x = 360$

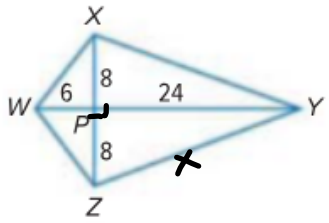
$200 + 2x = 360$

$2x = 160$

$x = 80$

b. If *WXYZ* is a kite, find *ZY*.

Because the diagonals of a kite are perpendicular, they divide *WXYZ* into four right triangles. Use the Pythagorean Theorem to find *ZY*, the length of the hypotenuse of right $\triangle YPZ$.



$\therefore x^2 = 24^2 + 8^2$

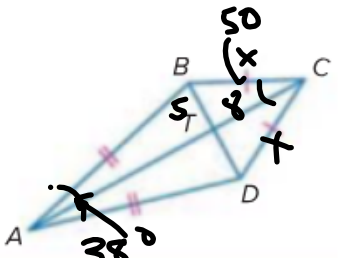
$x^2 = 576 + 64$

$\sqrt{x^2} = \sqrt{640}$ $x = \sqrt{640}$

Guided Practice

4A. If $m\angle BAD = 38$ and $m\angle BCD = 50$, find $m\angle ADC$.

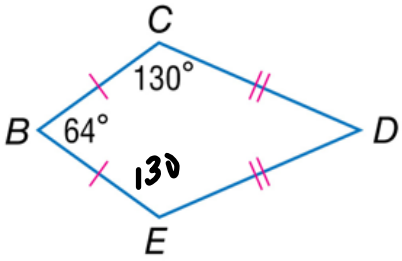
4B. If $BT = 5$ and $TC = 8$, find CD.



$50+38+2x=360$
-88
$2x=272$
2
$x=136$

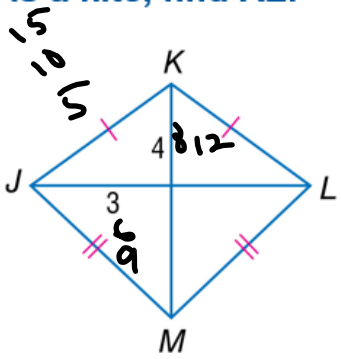
$x^2=5^2+8^2$
$x^2=25+64$
$x^2=89$
$x=\sqrt{89}$

A. If *BCDE* is a kite, find $m\angle CDE$.



$260+64+\angle D=360$
$324+\angle D$
-324
36

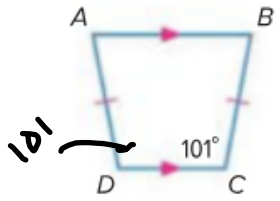
B. If *JKLM* is a kite, find *KL*.



5 10 15
12 24 36
13 26 39

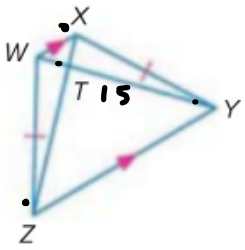
Find each measure.

1. $m\angle D$



101

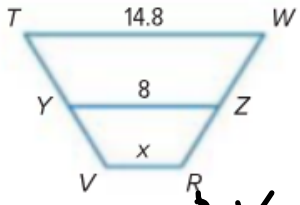
2. WT, if $ZX = 20$ and $TY = 15$



WT = 5

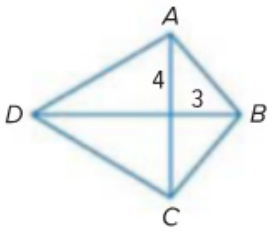
WY ≅ XZ

5. In the figure at the right, $\overline{YZ}$ is the midsegment of trapezoid $TWRV$. Determine the value of x.

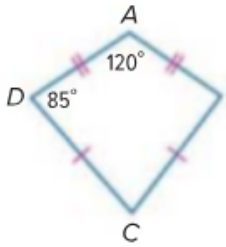


SENSE-MAKING If $ABCD$ is a kite, find each measure.

6. AB



7. $m\angle C$



$2 \cdot 8 = \frac{1}{2}(x + 14.8)$

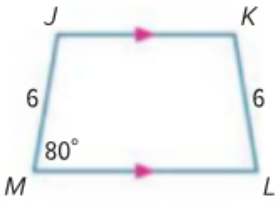
$16 = (x + 14.8$

$-14.8 \quad -14.8$

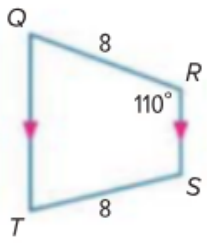
$1.2 = x$

Find each measure.

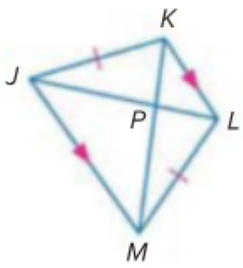
8. $m\angle K$



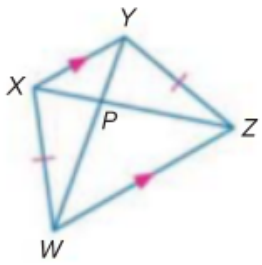
9. $m\angle Q$



10. JL, if $KP = 4$ and $PM = 7$



11 PW, if $XZ = 18$ and $PY = 3$



For trapezoid $QRTU$, V and S are midpoints of the legs.

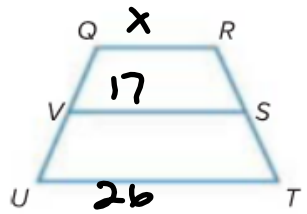
16. If $QR = 12$ and $UT = 22$, find VS.

17. If $QR = 4$ and $UT = 16$, find VS.

18. If $VS = 9$ and $UT = 12$, find QR.

19. If $TU = 26$ and $SV = 17$, find QR.

20. If $QR = 2$ and $VS = 7$, find UT.



8

$2 \cdot 17 = \frac{26+x}{2} \cdot 2$

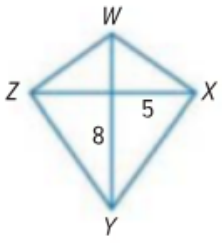
$34 = 26 + x$

$-26 \quad -26$

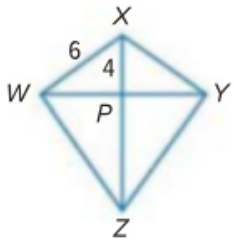
$x = 8$

SENSE-MAKING If $WXYZ$ is a kite, find each measure.

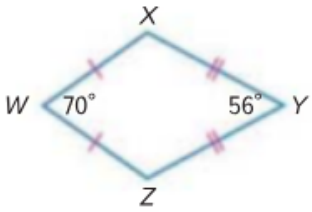
24. YZ



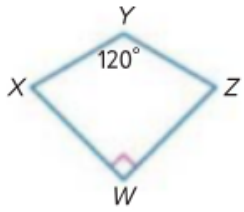
25. WP



26. $m\angle X$



27. $m\angle Z$

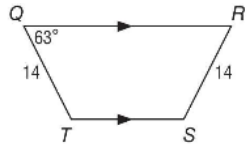


NAME _____ DATE _____ PERIOD _____

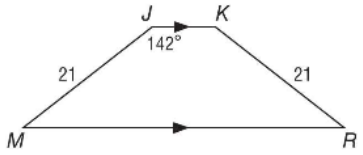
6-6 Skills Practice
Trapezoids and Kites

ALGEBRA Find each measure.

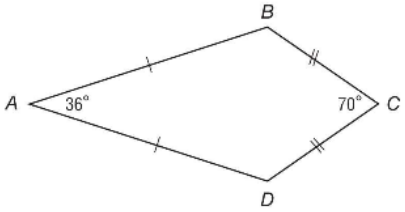
1. $m\angle S$



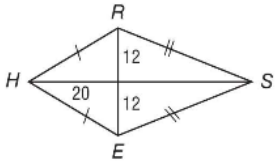
2. $m\angle M$



3. $m\angle D$

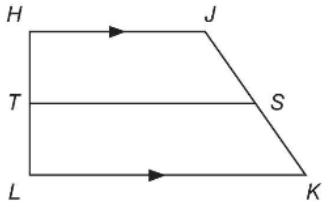


4. RH



ALGEBRA For trapezoid *HJKL*, *T* and *S* are midpoints of the legs.

5. If $HJ = 14$ and $LK = 42$, find TS.



6. If $LK = 19$ and $TS = 15$, find HJ.

7. If $HJ = 7$ and $TS = 10$, find LK.

8. If $KL = 17$ and $JH = 9$, find ST.

NAME _____ DATE _____ PERIOD _____

COORDINATE GEOMETRY ***EFGH*** **is a quadrilateral with vertices** ***E*****(1, 3),** ***F*****(5, 0),** ***G*****(8, –5),** ***H*****(–4, 4).**

9. Verify that *EFGH* is a trapezoid.

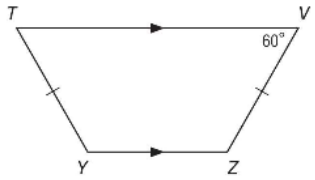
10. Determine whether *EFGH* is an isosceles trapezoid. Explain.

NAME _____ DATE _____ PERIOD _____

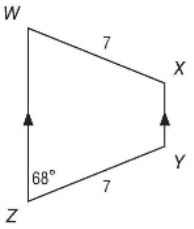
6-6 Practice
Trapezoids and Kites

Find each measure.

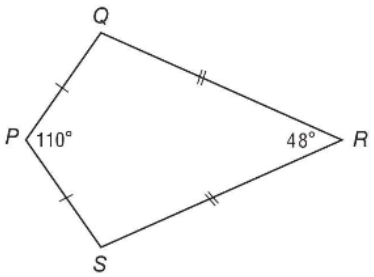
1. $m\angle T$



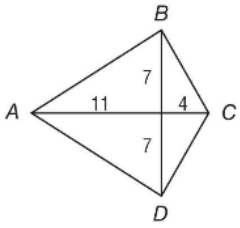
2. $m\angle Y$



3. $m\angle Q$

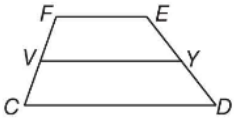


4. BC



ALGEBRA For trapezoid *FEDC*, *V* and *Y* are midpoints of the legs.

5. If $FE = 18$ and $VY = 28$, find CD.



6. If $m\angle F = 140$ and $m\angle E = 125$, find $m\angle D$.