Section 12.8 Counting Principle and Permutations

Ex : Selecting Balls without replacement

1. Two balls are selected without replacement (item cannot be selected more than once) from a bag that contains one red, blue, green, and orange ball.

a) Use the counting principle to determine the number of points.
4 choices for the first ball and 3 choices for the second = $4 \cdot 3 = 12$

Objective: In this section we will learn how to determine the number of different **ordered arrangements** of a set of objects.

Recall the Counting Principle -
Experiment 1 - M distinct ways
Experiment 2 - N distinct ways

Together M• N distict ways

We need three volunteer students.

Example: A store manager wants to display 5 different TV's in a showroom. How many different ways can they be arranged?

$5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

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Example 1
A password used to gain access to a computer account is to consist of two lower case letters followed by four digits. Determine how many different passwords are possible if

a) repetition of letters and digits is permitted

$26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 6{,}760{,}000$

b) repetition of letters and digits is not permitted.

$26 \cdot 25 \cdot 10 \cdot 9 \cdot 8 \cdot 7 = 3{,}276{,}000$

c) the first letter must be a vowel and the first digit cannot be a 0, and repetition of letters and digits is not permitted.

$5 \cdot 25 \cdot 9 \cdot 9 \cdot 8 \cdot 7 = 567{,}000$

Example 2

At Old Navy, a supply of T-shirts has just been received. The shirts come in the following colors: green, blue, white, yellow, and red. Billy Bragg, the floor manager, decides to display one of each color shirt in a row on a shelf.

a) In how many different ways can he arrange the five different color shirts on a shelf?

$5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

b) If he wants to place the blue shirt in the middle, in how many different ways can he arrange the shirts?

$4 \quad 3 \quad 1 \quad 2 \quad 1 = 24$

c) If Billy want the white shirt to be the first shirt and the blue shirt to be the last shirt, in how many different ways can he arrange the shirts?

$1 \cdot 3 \cdot 2 \cdot 1 \cdot 1 = 6$

PROBLEM SOLVING

21. *ATM Codes* To use an automated teller machine, you generally must enter a four-digit code, using the digits 0–9. How many four-digit codes are possible if repetition of digits is permitted?

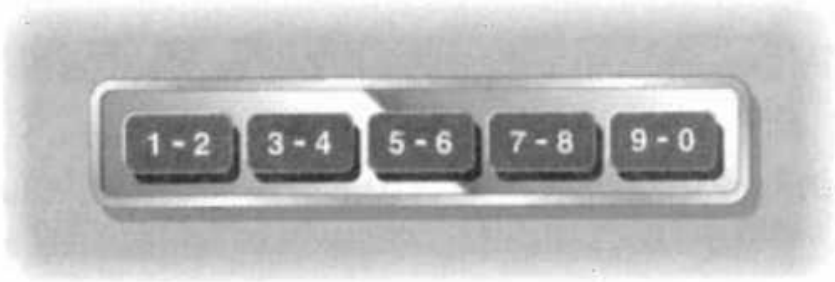
$10 \cdot 10 \cdot 10 \cdot 10 = 10{,}000$

23. *Passwords* Assume that a password to log onto a computer account is to consist of three letters followed by two digits. Determine the number of possible passwords if

a) repetition is not permitted. $26 \cdot 25 \cdot 24 \cdot 10 \cdot 9 = 1{,}404{,}000$

b) repetition is permitted. $26 \cdot 26 \cdot 26 \cdot 10 \cdot 10 = 1{,}757{,}600$

25. *Car Door Locks* Some doors on cars can be opened by pressing the correct sequence of buttons. A display of the five buttons by the door handle of a car follows.*



The correct sequence of five buttons must be pressed to unlock the door.

$5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 = 3125$

a) If the same button may be pressed consecutively, how many possible ways can the five buttons be pressed (repetition is permitted)?

b) If five buttons are pressed at random, determine the probability that a sequence that unlocks the door will be entered.

$\frac{1}{3125}$

27. *License Plate* A license plate is to have five uppercase letters or digits. Determine the number of license plates possible if repetition is permitted and if any position can contain either a letter or digit with the exception that the first position cannot contain the letter *O* or the number 0.

$36 \cdot 36 \cdot 36 \cdot 36 \cdot 36$

$= 57{,}106{,}944$

$26 + 10 = 36$

30. *Geometric Shapes* Consider the five figures shown.

In how many different ways can the figures be arranged

a) from left to right?

b) from top to bottom if placed one under the other?

c) from left to right if the triangle is to be placed on the far right?

d) from left to right if the circle is to be placed on the far left and the triangle is to be placed on the far right?

31. ***Arranging Pictures*** The six pictures shown are to be placed side by side along a wall.

In how many ways can they be arranged from left to right if

a) they can be arranged in any order?

b) the bird must be on the far left?

c) the bird must be on the far left and the giraffe must be next to the bird?

d) a four-legged animal must be on the far right?

A **Permutation** is any ordered arrangement of a given set of objects.

example:

Larry, Moe, Curly Curly, Larry, Moe

The same three names yet two different ordered arrangements (permutaion)

How many different arrangements (permutations) of three object.

$3! = 3 \cdot 2 \cdot 1 = 6$

When determining the number of permutations possible, we assume that repetition of an item is not permitted.

Example: Larry, Moe, Moe

Example 3

In how many ways can 7 children be arranged in a line?