Section 6-5 The Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$ax^2 + bx + c = 0$$

Solve using the quadratic formula

$x^2 - 12x = 28$

$-28 \quad -28$

$x^2 - 12x - 28 = 0$

$a = 1$
$b = -12$
$c = -28$

$x = \frac{-(-12) \pm \sqrt{-12^2 - 4(1)(-28)}}{2(1)}$

$= \frac{12 \pm \sqrt{256}}{2}$

$= \frac{12 \pm 16}{2}$

$x = \frac{12 + 16}{2} = 14 \quad x = \frac{12 - 16}{2} = -2$

$2x^2 - 8x - 90 = 0$

$x = \frac{-(-8) \pm \sqrt{-8^2 - 4(2)(-90)}}{2(2)}$

$x = \frac{8 \pm \sqrt{784}}{4}$

$x = \frac{8 \pm 28}{4}$

$x = \frac{8 + 28}{4} = 9 \quad x = \frac{8 - 28}{4} = -5$

Solve by using the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$9x^2 - 12x + 4 = 0$$

$$x = \frac{-(-12) \pm \sqrt{-12^2 - 4(9)(4)}}{2(9)}$$

$$x = \frac{12 \pm \sqrt{0}}{18} = \frac{2}{3}$$

$$2x^2 + 16x + 33 = 0$$

$$x = \frac{-16 \pm \sqrt{16^2 - 4(2)(33)}}{2(2)}$$

$$x = \frac{-16 \pm \sqrt{-8}}{4}$$

$\sqrt{-8}$

$i\sqrt{4}\sqrt{2}$

$2i\sqrt{2}$

$$x = \frac{-16 \pm 2i\sqrt{2}}{4}$$

$$x = \frac{-8 \pm 1i\sqrt{2}}{2}$$

Solve

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$2x^2+4x-5=0$$

$$x=\frac{-4\pm\sqrt{4^2-4(2)(-5)}}{2(2)}$$

$$x=\frac{-4\pm\sqrt{56}}{4}$$

$\sqrt{4}\sqrt{14}$

$2\sqrt{14}$

$$x=\frac{-4\pm 2\sqrt{14}}{4}$$

$$x=\frac{-2\pm\sqrt{14}}{2}$$

$$15x^2-7x-4=0$$

$$x=\frac{-(-7)\pm\sqrt{-7^2-4(15)(-4)}}{2(15)}$$

$$x=\frac{7\pm\sqrt{289}}{30}$$

$$x=\frac{7\pm 17}{30}$$

$$x=\frac{7+17}{30}=\frac{24}{30}$$

$\frac{4}{5}$

$$x=\frac{7-17}{30}=-\frac{10}{30}$$

$-\frac{1}{3}$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solve

$$2x^2 + 5x + 6 = 0$$

$$x = \frac{-5 \pm \sqrt{5^2 - 4(2)(6)}}{2(2)}$$

$$x = \frac{-5 \pm \sqrt{-23}}{4}$$

$$x = \frac{-5 \pm i\sqrt{23}}{4}$$

$$-5x^2 + 8x - 1 = 0$$

Solve

$$7x^2 + 6x = -2$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The expression $b^2 - 4ac$ is called the **discriminant**.

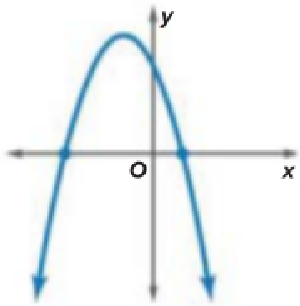
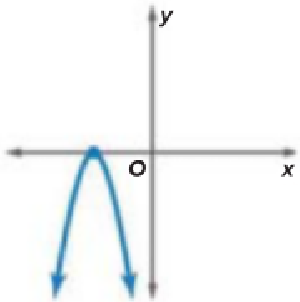
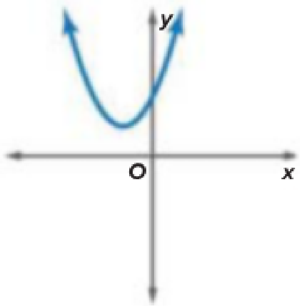
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \leftarrow \text{discriminant}$$

The value of the discriminant can be used to determine the number and type of roots of a quadratic equation. The table on the following page summarizes the possible types of roots.

The discriminant can also be used to confirm the number and type of solutions after you solve the quadratic equation.

Key Concept Discriminant

Consider $ax^2 + bx + c = 0$, where a, b, and c are rational numbers and $a \neq 0$.

Value of Discriminant	Type and Number of Roots	Example of Graph of Related Function
$b^2 - 4ac > 0$; $b^2 - 4ac$ is a perfect square.	2 real, rational roots	
$b^2 - 4ac > 0$; $b^2 - 4ac$ is *not* a perfect square.	2 real, irrational roots	
$b^2 - 4ac = 0$	1 real rational root	
$b^2 - 4ac < 0$	2 complex roots	

Example 5 Describe Roots

Find the value of the discriminant for each quadratic equation. Then describe the number and type of roots for the equation.

a. $7x^2 - 11x + 5 = 0$

b. $x^2 + 22x + 121 = 0$

Guided Practice

5A. $-5x^2 + 8x - 1 = 0$

5B. $-7x + 15x^2 - 4 = 0$

Complete parts a and b for each quadratic equation.

a. Find the value of the discriminant.

b. Describe the number and type of roots.

10. $3x^2 + 8x + 2 = 0$

11. $2x^2 - 6x + 9 = 0$

12. $-16x^2 + 8x - 1 = 0$

13. $5x^2 + 2x + 4 = 0$

Concept Summary Solving Quadratic Equations

Method	Can be Used	When to Use
graphing	sometimes	Use only if an exact answer is not required. Best used to check the reasonableness of solutions found algebraically.
factoring	sometimes	Use if the constant term is 0 or if the factors are easily determined. **Example** $x^2 - 7x = 0$
Square Root Property	sometimes	Use for equations in which a perfect square is equal to a constant. **Example** $(x - 5)^2 = 18$
completing the square	always	Useful for equations of the form $x^2 + bx + c = 0$, where b is even. **Example** $x^2 + 6x - 14 = 0$
Quadratic Formula	always	Useful when other methods fail or are too tedious. **Example** $2.3x^2 - 1.8x + 9.7 = 0$