Then

You determined whether quadrilaterals were parallelograms and/or rectangles.

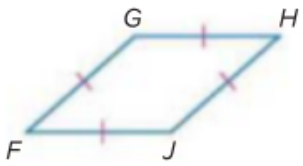
Now

- Recognize and apply the properties of rhombi and squares.
- Determine whether quadrilaterals are rectangles, rhombi, or squares.

Section 6.5 Squares and Rhombi

1 Properties of Rhombi and Squares

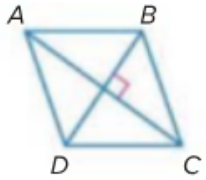
A **rhombus** is a parallelogram with all four sides congruent. A rhombus has all the properties of a parallelogram and the two additional characteristics described in the theorems below.



Theorems Diagonals of a Rhombus

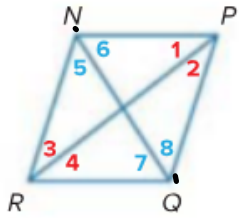
6.15 If a parallelogram is a rhombus, then its diagonals are perpendicular.

Example If $\square ABCD$ is a rhombus, then $\overline{AC} \perp \overline{BD}$.



6.16 If a parallelogram is a rhombus, then each diagonal bisects a pair of opposite angles.

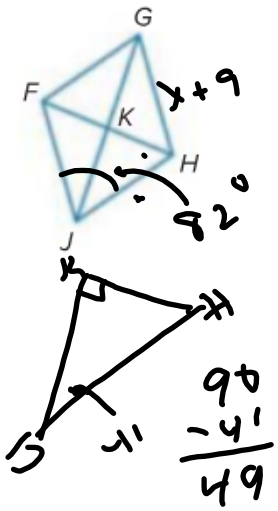
Example If $\square NPQR$ is a rhombus, then $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$, $\angle 5 \cong \angle 6$, and $\angle 7 \cong \angle 8$.



Example 1 Use Properties of a Rhombus

The diagonals of rhombus $FGHJ$ intersect at K. Use the given information to find each measure or value.

a. If $m\angle FJH = 82$, find $m\angle KHJ$. $= \frac{98}{2} = 49^\circ$



b. **ALGEBRA** If $GH = x + 9$ and $JH = 5x - 2$, find x.

$x + 9 = 5x - 2$

$11 = 4x$

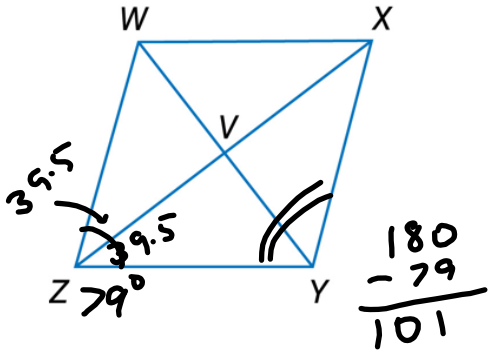
$x = 1\frac{1}{4}$

Refer to rhombus $FGHJ$ above.

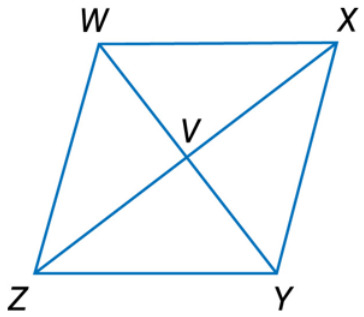
1A. If $FK = 5$ and $FG = 13$, find KJ.

1B. ALGEBRA If $m\angle JFK = 6y + 7$ and $m\angle KFG = 9y - 5$, find y.

A. The diagonals of rhombus *WXYZ* intersect at *V*. If $m\angle WZX = 39.5°$, find $m\angle ZYX$.



B. ALGEBRA The diagonals of rhombus *WXYZ* intersect at *V*. If $WX = 8x - 5$ and $WZ = 6x + 3$, find *x*.

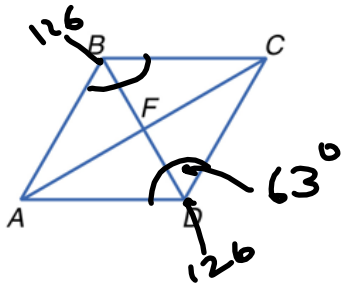


$8x - 5 = 6x + 3$

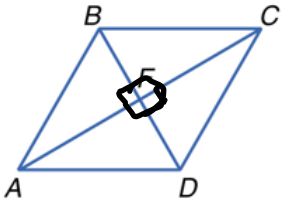
$2x = 8$

$x = 4$

A. ***ABCD*** **is a rhombus. Find** $m\angle CDB$ **if** $m\angle ABC = 126$.



B. ***ABCD*** **is a rhombus. If** $BC = 4x - 5$ **and** $CD = 2x + 7$, **find** x.

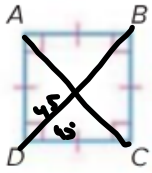


$4x - 5 = 2x + 7$

$2x = 12$

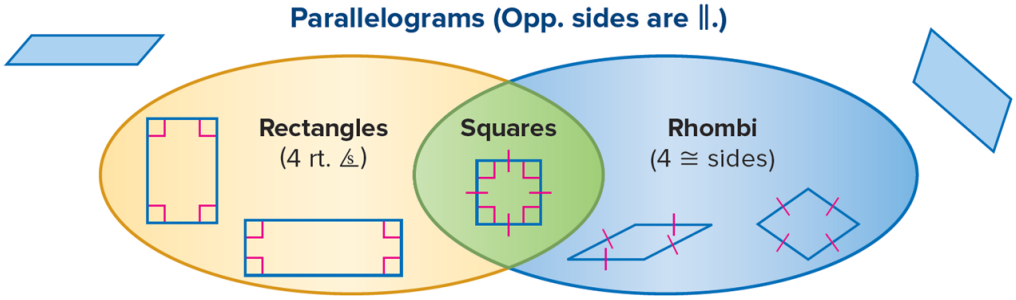
$x = 6$

A **square** is a parallelogram with four congruent sides and four right angles. Recall that a parallelogram with four right angles is a rectangle, and a parallelogram with four congruent sides is a rhombus. Therefore, a parallelogram that is both a rectangle and a rhombus is also a square.



Square *ABCD*

Concept Summary Parallelograms



All of the properties of parallelograms, rectangles, and rhombi apply to squares. For example, the diagonals of a square bisect each other (parallelogram), are congruent (rectangle), and are perpendicular (rhombus).

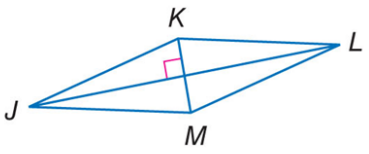
2 Prove that Quadrilaterals are Rhombi or Squares

The theorems below provide conditions for rhombi and squares.

Theorems Conditions for Rhombi and Squares

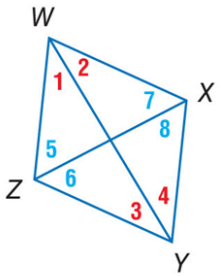
6.17 If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus. (Converse of Theorem. 6.15)

Example If $\overline{JL} \perp \overline{KM}$, then $\square JKLM$ is a rhombus.



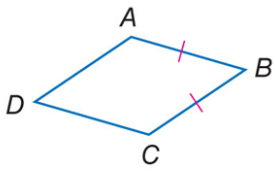
6.18 If one diagonal of a parallelogram bisects a pair of opposite angles, then the parallelogram is a rhombus. (Converse of Theorem. 6.16)

Example If $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$, or $\angle 5 \cong \angle 6$ and $\angle 7 \cong \angle 8$, then $\square WXYZ$ is a rhombus.



6.19 If one pair of consecutive sides of a parallelogram are congruent, the parallelogram is a rhombus.

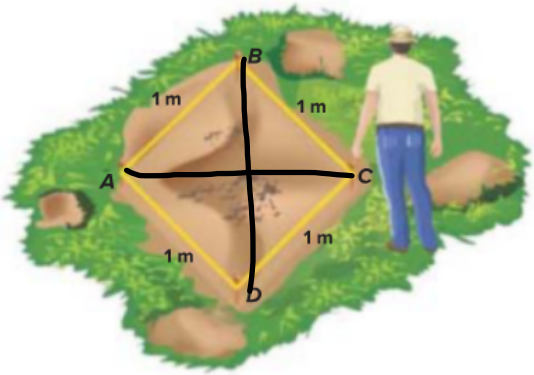
Example If $\overline{AB} \cong \overline{BC}$, then $\square ABCD$ is a rhombus.



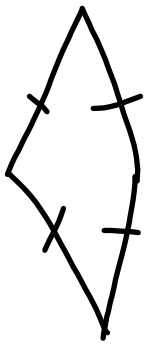
6.20 If a quadrilateral is both a rectangle and a rhombus, then it is a square.

Real-World Example 3 Use Conditions for Rhombi and Squares

ARCHAEOLOGY The key to the successful excavation of an archaeological site is accurate mapping. How can archaeologists be sure that the region they have marked off is a 1-meter by 1-meter square?

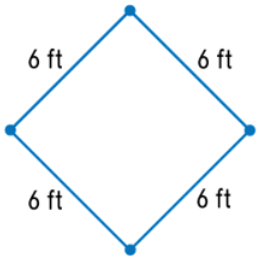


diagonals are = then □

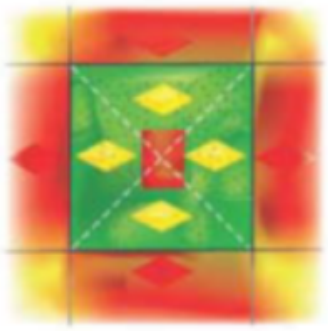


Real-World Example 3 Use Conditions for Rhombi and Squares

GARDENING Hector is measuring the boundary of a new garden. He wants the garden to be square. He has set each of the corner stakes 6 feet apart. What does Hector need to know to make sure that the garden is square?



3. **QUILTING** Kathy is designing a quilt with blocks like the one shown.



A. If she marks the diagonals of each yellow piece and determines that each pair of diagonals is perpendicular, can she conclude that each yellow piece is a rhombus? Explain.

B. If all four angles of the green piece have the same measure and the bottom and left sides have the same measure, can she conclude that the green piece is a square? Explain.

Sachin has a shape he knows to be a parallelogram and all four sides are congruent. Which information does he need to know to determine whether it is also a square?

A. **The diagonal bisects a pair of opposite angles.**

B. **The diagonals bisect each other.**

C. **The diagonals are perpendicular.**

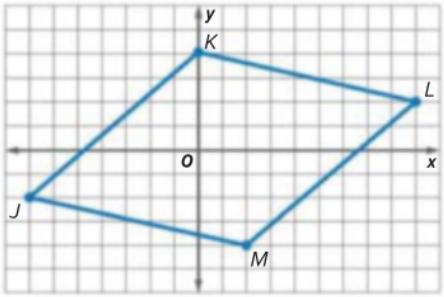
D. **The diagonals are congruent.**

Example 4 Classify Quadrilaterals Using Coordinate Geometry

COORDINATE GEOMETRY **Determine whether $\square JKLM$ with vertices $J(-7, -2)$, $K(0, 4)$, $L(9, 2)$, and $M(2, -4)$ is a *rhombus*, a *rectangle*, or a *square*. List all that apply. Explain.**

Understand Plot and connect the vertices on a coordinate plane.

It appears from the graph that the parallelogram has four congruent sides, but no right angles. So, it appears that the figure is a rhombus, but not a square or a rectangle.



Plan If the diagonals of the parallelogram are congruent, then it is a rectangle. If they are perpendicular, then it is a rhombus. If they are both congruent and perpendicular, the parallelogram is a rectangle, a rhombus, and a square.

Solve **Step 1** Use the Distance Formula to compare the diagonal lengths.

Step 2 Use the Slope Formula to determine whether the diagonals are perpendicular.

Guided Practice

4. **Given** $J(5, 0)$, $K(8, -11)$, $L(-3, -14)$, $M(-6, -3)$, determine whether parallelogram $JKLM$ is a *rhombus*, a *rectangle*, or a *square*. List all that apply. Explain.

ALGEBRA **Quadrilateral $ABCD$ is a rhombus. Find each value or measure.**

7. If $AB = 14$, find BC.

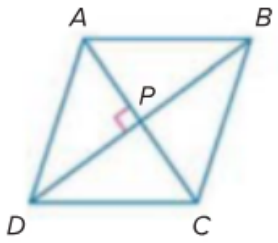
8. If $m\angle BCD = 54$, find $m\angle BAC$.

9. If $AP = 3x - 1$ and $PC = x + 9$, find AC.

10. If $DB = 2x - 4$ and $PB = 2x - 9$, find PD.

11 If $m\angle ABC = 2x - 7$ and $m\angle BCD = 2x + 3$, find $m\angle DAB$.

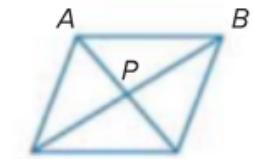
12. If $m\angle DPC = 3x - 15$, find x.



COORDINATE GEOMETRY Given each set of vertices, determine whether $\square JKLM$ is a *rhombus*, a *rectangle*, or a *square*. List all that apply. Explain.

19. $J(-4, -1)$, $K(1, -1)$, $L(4, 3)$, $M(-1, 3)$

ABCD is a rhombus. If $PB = 12$, $AB = 15$, and $m\angle ABD = 24$, find each measure.



23. AP

24. CP

25. $m\angle BDA$

26. $m\angle ACB$